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Dynamic and instability analysis of single walled carbon nanotubes with geometrical imperfections resting on elastic medium in a magneto-thermally-electrostatic environment with impact of casimir force using homotopy perturbation method

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ABSTRACT

The discovery of carbon nanotubes (CNTs) has renewed a major chapter in the field of physics, chemistry, mechanics and materials science owing to their high-quality possession of: excellent tensile strength, high conductivity, high aspect ratio, thermally stable and high chemical stability. This work studies the dynamic and instability analysis of single walled carbon nanotube with geometrical imperfection resting on elastic medium in a magneto-thermally-electrostatic environments with impact of Casimir force. However, Eringen nonlocal theory and Hamilton principles are used to develop the nonlinear governing partial differential equations of motions and the governing equations of motion is converted into a duffing equation using Galerkin decomposition method and subsequently, the duffing equation is solve using Homotropic Perturbation Method (HPM), where dynamic responses are obtained. The results obtain depicted that, the effects of magnetic term, thermal term and Pasternak type foundation on dimensionless amplitude-frequency ratio for fixed-fixed and fixed-simple supports make the investigation novelty as it can be used as reference in future study. Finally, the deflection curves show how the compression zone is augmented using Casimir and electrostatic forces and the results obtained show reasonable accuracy.

KEYWORDS

Single-walled carbon nanotubes, Magnetic, Thermal, Homotopic perturbation method, Elastic foundation, Casimir force, Electrostatic force

1. INTRODUCTION

The discovery of carbon nanotubes (CNT) by Iijima has renewed a major chapter in the field of mechanics, physics, chemistry and materials science owing to their possession of high strength, stiffness etc. as tomorrow society will be shaped with the potential use of nanoscale structures. Because of these novelty, nonlinear vibrational analysis of carbon nanotubes has been continuously subjected to several investigation to unraveled its dynamically responses under various end support conditions.

Ravi kumar B. (2017) studied vibrational response of doublewalled carbon nanotubes (DWCNTs) for several end supports using differential transform method (DTM).

Payam Soltani et al. (2011) analyzed transverse vibrational of singlewalled carbon nanotubes (SWCNT) embedded in a spring foundation for various end supports using perturbation method of multi-scales.

Ghasemi et al. (2015) used variation iteration method (VIM) and energy balance method (EBM) to study nonlinear Vibrational response of carbon nanotubes with Nonlocal Timoshenko Beam Theory and the results obtained from the above analytical methods was comparison with Runge-Kutta order four with high agreed solutions.

Fu et al. (2006), investigated nonlinear free vibrations of embedded multi-walled carbon nanotubes with intertube radially displacement using increment harmonic balance method. Subsequently, the amplitude frequency response curves for both nonlinear free vibrations of single-walled and double-walled carbon nanotubes was obtained and discussed extensively.

Ender and Hamed (2014) studied nonlinear free vibrational analysis of slightly curve double walled carbon nanotube embedded in an elastic spring using differential quadrature method (DQM) to discretize the partial differential equations of motions with different support conditions with observation that support conditions have significant effects on the natural frequencies of double-walled carbon nanotubes including multiple solutions.

Wu et al. (2018) studied nonlinear free vibrations analysis of multiwall carbon nanotube embedded in an elastic spring using a direct iterative approach.

Venkatraman and Suji (2022) presented flow-induced nonlinear vibrational behaviour of single walled carbon nanotube under elastic foundation using homotopic perturbation method and elliptical functions for exact solution of differential equations. The obtained results of both HPM and exact solutions are compared for linear and nonlinear frequencies.

Musab and Shaymaa (2016) studied effects of multi-walled carbon nanotubes on the behavioral of reinforced concrete beam under monotonic loads. Moreso seven beam was loaded to failure under the effects of two concentrated loadings, only reference beams were reinforced with steel reinforcement and rest six-beam was reinforced with shorter and longer Multi-Walled Carbon Nanotubes at different concentrated interval of 0.03% and 0.06% by weight of dry Cementous in addition to steel reinforcement.

Khader et al. (2014) analyzed nonlinear dynamic vibration analysis of multi-walled carbon nanotubes embedded in elastic medium using homotopy analysis method (HAM). Subsequently, the amplitude frequency curves for larger amplitude vibrations for single, double and triple-walled carbon nanotubes are obtained and compared with Adomian decomposition method.

Tai-Ping (2013) performed dynamical analysis of nonlinear vibrational of fluid conveying doublewalled carbon-nanotubes using perturbation methodology. Furthermore, the nonlinear govern equation of the fluid conveying decomposed into set of partial differential equations and galerkin decomposition technique is use to analysis the resulting nonlinear equations successively.

Wang and Wang (2022) studied vibrational analysis carbon-nanotubes fixed in elastic foundation using new novelty method of Hamiltonian Based Method to analyzed frequencies property of nonlinear vibrations. Furthermore, effective and reliability of new novelty method was verify through numerically solutions and results obtained are expected to be useful for future investigation of nonlinear vibrational analysis.

Valipour et al. (2016) characterized nonlinear vibrational of embedded single walled carbon-nanotube on Pasternak typical model using parameterised perturbations method to solved the resulting partial equation of motions and it depicts by increasing the Winkler-type constant, the nonlinear frequencies decrease as well.

Hossein and Hemmatnezhad (2011) applied homotopy perturbation method to solve nonlinear frequency of multi-walled carbon-nanotubes fixed in an elastic foundation with several supported ends as well as obtained solutions for amplitude frequency curves for larger-amplitude vibration of singlewalled, doublewalled, and triplewalled carbon-nanotubes. Solutions obtained from analytical and open literation are compared and there are in higher agreement as a new benchmark for future reference.

Hossain and Lellep (2021) modelled dynamic behaviour of SWCNTs partially fixed into elasticity soil foundation by Euler Bernoulli beam and nonlocal theories of elasticity using semi-analytic method to solve the resulting governed partial differential equations of motion. Furthermore, effect of temperature, coefficient of elasticities medium and nonlocal parameter of dynamical behaviour of SWCNTs was mainly focus and was concluded as the results of the simulations conducted have significant effect on natural frequency of nanotube structures under investigation.

Chuanyong (2007) employed Donnell equations of cylindrical shells to investigate nonlinear vibrational behavior of a multi-walled carbon nanotubes under elastic multilayer shell model considering van der Waals interaction with negligible between the inner-outer tubes in of the proposed model and th van der Waals interaction between each layer show that its presence forces can strongly influence the buckling and nonlinear vibrational of the multiwalled carbon nanotubes.

Tai-Ping and Quey-Jen (2018) investigated effects of chaotic behaviors of single walled carbon nanotube for both linear-nonlinear damping using the Hamilton's principle. Galerkin's decompactions is use to simplify the integro partial differential equations of motion into nonlinear dimensionless governed equation of motions. Subsequently, forced Duffing equation was obtained. Also, they studied variations of Lyapunov exponents of the single walled carbon nanotube with damping-harmonic forcing amplitudes. Lyapunov exponent shows that chaotical motions of the single walled carbon nanotube occurred when both the amplitude of the periodical excitations exceeds expected values, smaller linear and nonlinear damping's.

Chowdhury et al. (2010) investigated vibrational properties of armchair-zigzag SWCNT using molecular mechanic to obtained natural-frequencies of vibrations and there modes. Furthermore, simulation of four and three different types of zigzag and armchair SWCNTs such as (5, 0), (6, 0), (8, 0), (10, 0) and (3, 3), (4, 4), (6, 6) are carried. Moreso, the results obtained show that as natural frequency decrease, aspect ratios increase and there all follow similarity trends with result of past studies for CNT using same methods.

Azrar et al. (2014) modelled flow induces nonlinear free vibrations of SWCNT using von Kármán Eringen's nonlocal elasticity and geometric nonlinearity theory. Furthermore, partial differential governing equations of motion with boundary conditions was derived using the Hamilton's principle and the equation of motion is solved using Galerkin's technique. Moreso, with use of this principles, the following can be investigated, small scale parameter, fluid tube interaction effects and instabilities induce by the fluid-velocity. Moreso, critical fluid velocity, frequency-amplitude relationships, flutter and divergence instabilities type with time responses was acquired through the approached methodology.

Abdelkadir et al. (2016) investigated bending vibrational and dynamical analysis of singlewalled carbon-nanotube using nonlocal elastic theory. Furthermore, they also compute natural-frequencies and mode shapes of the single walled carbon nanotubes using semi-analytical technique of differential quadrature (DQ). Moreso, the results obtained are in agreement with numerical method of exact solution.

Farhand (2014) presented gradient elasticity of shell for free vibrational analysis of singlewalled carbon nanotubes under Winkler and Pasternak medium. Furthermore, they investigated effect of the length-scale parameters, aspects ratio of singlewalled carbon-nanotube and spring parameters on the fundamental's frequency for several values' half axial and circumferential wave numbers. Moreso, natural-frequencies acquired by methodology shows effect of size dependent properties. Finally, its concluded as well that a continuum modeled enriched with higher order inertia terms are proposed as an alternative to continuum descriptions acquired through classical elasticity theories.

Matteo et al. (2014) considered Low-frequency vibrational analysis of SWCNTs with different boundary conditions using two approached of semi-analytical method. Furthermore, first method, used Rayleigh Ritz method, of doubly series expansions in term of Chebyshev polynomial-harmonic functions where, free and clamped edges was analyzed as the method is partially numeric while other method was basically on thin shell theories and its aims is to obtained analytical solutions for future usage in nonlinearity fields. The results obtained from this method was validated with experimental values of molecular dynamical data with FEA from literature.

Soltani et al. (2011) investigated nonlinear free-force vibrational analysis of singlewalled carbon-nanotube considering simply supported ends using von Karman's geometrical nonlinearity theory. Furthermore, Galerkin's techniques is used to discretized the govern PDEs into an ODE of motions. Thereafter, methods of averaging is applied to solve nonlinear vibrations of following zig-zag single-walled carbon nanotubes as (10, 0), (20, 0) and (30, 0) in analytical calculations. Moreso, the following was investigated, there are; different aspect ratios, effect of nonlinear parameters, different circumferential and longitudinal half-wave numbers. In addition, freed and forced motion due to harmonical excitations was investigated in the analysis. It depicted, that (30, 0) zig-zag single-walled carbon nanotube exhibit lesser nonlinear behavior like other carbon nanotube for constantly aspect ratio. It was concluded from the result obtained that for smaller value of aspect ratios, vibrational behaviors are softening-type for lower amplitudes and it is hardening-type for larger amplitude and for larger values of aspect ratio, vibrational behaviors are hardening-type for every amplitude.

Hussein et al. (2015) studied nonlinear vibrational analysis of singlewalled carbon nanotubes embedded in Kelvin-Voigt elastic-medium, considering an elastic Euler Bernoulli beam with von- Kármán type geometric nonlinear. Thereafter, Hamilton's principle is use to drive the PDE as governing equation of motions and nonlocal elastic fields theory was used to introduced smaller-scale effects into the equations of motion. Galerkin's decomposition method was used to reduce the resulting PDEs into an ODE and was subsequently solve using asymptotic perturbations method known as Krylov Bogolubov Mitropolskij techniques. Moreso, other semi-analytical technques are formulated to handle frequency and displacement and the results obtained from the analysis revealed that using simple-simple end supports, the effect of residually stresses, viscoelastical foundation and amplitude are adequately discussed in details.

Maria et al. (2021) considered free vibrations of tapered beams modelled nonuniformly SWCNTs, also known as nanocones. Furthermore, the nanocones beams is fixed in one ends and elastically restrain at other ends with concentrated mass attached. Moreso, nonlinear governing equation of motion with boundary conditions and nonlocal small-scale effects was considered in the formulation and differential quadrature method was used to compute natural frequencies, tapered ratio coefficient, smaller-scale parameters added mass on first natural frequency. Also, a numerical solution was used to verify and validate the proposed techniques with the analytical solutions and from the results obtained are in goods agreement. Finally, it was concluded as follow, these methods can be applied to other types of ends supports.

Mu'tasim et al. (2017) studied and investigated nonlinear free vibrational analysis and frequencies veering of SWCNTs based on Eringen nonlocal elasticity theories. Furthermore, the governed nonlinear equations of motion was modeled using Euler Bernoulli Beams and Hamilton's principles, the modeled accounted for non-local elasticity, geometrical initial rise/imperfections and the effects of the axially forces induce by the mid plane stretch. Moreso, this method of multiple-scales (MMS) was use to solve resulting nonlinear equation to acquire natural-frequencies of various rise/imperfection amplitude and the results are shown in dimensionless and discussed in detailed.

Milad and Aminikhah (2010) studied nonlinear vibration analysis of MWCNT in elastic foundation using variational iterations method. Furthermore, the governing equations of motions was developed with multiple beams to couple the following, single, double, triple and multiple-walled carbon nanotube. Moreso, effects of vibrational characteristic of nanotubes geometric parameters are considered and the results obtained was compared with earliest literature findings and there are all in good agreements.

Soltani et al. (2014) investigated nonlinear vibrational analysis of fluid-filled (SWCNT) with pinned-pinned supports. Furthermore, effect of smaller-scale parameters are integrated with the aid nonlocal theories, Galerkin's decomposition method was introduced by discretizing the resulting governing of PDE into a more solvable ODE of motion thereafter, method of averaging analytical solution was applied on the ODE. The single walled carbon nanotube is presumed to be filled by water-fluid with assumed to be ideals non-compression, non-rotational and inviscid-type. Again, model was solved by the semi-analytical approach and the effect of nonlocal parameters was considered during simulation.

Askari et al. (2013) carried out nonlinear dynamic analysis of SWCNTs resting on Pasternak-type medium using Euler Bernoulli beams and Eringen nonlocal elastic theories under pinned-pinned supports boundaries condition. Furthermore, the governing equations of motion was developed using above stated theories and Galerkin decomposition method is applied on the resulting nonlinear PDE of motion and ODE was obtained for the SWCNTs. Thereafter, homotopic analysis method (HAM) is employed to analyzed the nonlinear natural-frequency. Again, simulation was conducted on some the parameters. In conclusion, numerical parametric study was conducted to ascertained if the semi-analytical and numerical results obtained agreed.

Smirnov et al. (2016) presented nonlinear dynamical analysis of single walled carbon nanotube of lower energy non-stationary. Furthermore, a newer phenomenon of intense energy-exchange between various part of carbon nanotubes and weaker energies localized in excited parts of carbon nanotube is analyzed and forecasted in the framework of the continuum shell theories. Moreso, clarification of their origin was means of conception of Limiting-Phase-Trajectory and the analyzed results and confirmation of molecular dynamical simulation of pinned-pinned ends support carbon nanotubes.

Eshraghi et al. (2016) considered imperfections sensitivity of larger amplitude vibrations of curved SWCNTs. Furthermore, the carbon-nanotubes is model like Timoshenko nanobeam with curved shape for initial geometrical imperfections terms in displacements field. Moreso, geometrical nonlinearity of von Kármán-type and Eringen non-local elasticity theory are employed to derived governing partial differential equation of motions of nanobeam. Thereafter, spatial discretized method was applied on the obtained nonlinear PDE of governing equation of motion and its associated ends supported conditions was conducted using differential quadrature (DQ) Again, values and location effects of the geometrical imperfections and Erigen non-local small-scale parameters on nonlinear frequency-ratio and imperfections sensitivities of curve single walled carbon-nanotubes for different end condition are studied and obtained results depict that the geometrical imperfections played significant roles in the nonlinear vibrational characteristics of curved single-walled carbon nanotube.

Rajabi et al. (2018) examined size dependent nonlinear vibrational analysis of Euler Bernoulli nano-beams acting upon by a moving harmonical load travelling with variables velocities. Furthermore, pinned-pinned supported ends, the Galerkin discretization procedure are used to convert PDEs of motion to an ODEs. Thereafter, multistage linearization technique (MLT) was use to solved the resulting ordinary differential equations accordingly. Again, effect of the Eringen nonlocal, material-length scale parameter, velocities, acceleration and excitations frequency of the moving harmonical loads on the nonlinear dynamical behavior of nano-beams was studied appropriately.

Saadatnia (2021) investigated nonlinear vibrational analysis of a curved-piezoelectric-layered CNTs resonators. Furthermore, with application of energy method and Hamiltonian principle couple with Eringen nonlocal theories, nonlinear partial differential equations of motion was modeled. Thereafter, partial differential equations of motion was reduced to Duffing equation after Galerkin decomposition method and multiple scales methods (MSM) of analysis was used to solved the resulting Duffing equation. Several resonances condition are studied including primaries and parametric resonance as well as frequency response are achieved with steady-state motions. Again, effect of several parameter like, piezoelectric thickness, apply voltage and structural curvatures on dynamic responses was studied. The obtained results depicted that, apply harmonic voltages to the piezoelectric layer could result to a parametric resonance in structural vibrations together with apply harmonic point loads to the structures could result to primary frequency in vibrational response. Moreso, quadratic and cubic curves of structural curvatures was also considered, it was finds that the wave and curved shape parameter could tune the nonlinear hardening-softening behavioral of any systemic and a specifically curved shape, the vibration responses to behave in similar way like that of linear systems. The investigation should extend towards design of curve piezoelectric nanoresonators in small-scale sensing and actuation system.

Fatahi-Vajari and Azimzadeh (2020) studied nonlinear coupled radial axial vibrational analysis of SWCNTs using numerical methods. Thereafter, nonlocal doublet mechanics (DM) theory was used to obtained nonlinear governing equations of motions and the equation of motion was solved using Homotopy perturbation method (HPM) which was subsequently use to determine nonlinear natural-frequencies of coupled radial-axial vibrational mode and found that its cumbersome coupling two vibrational modes of a systems due to their ends conditions, different in vibration mode shapes and other geometrically simulation parameters are all considered in details. Also, maximum vibrational velocity and end conditions play important roles in single walled carbon nanotubes vibrational response though linear, nonlinear natural frequencies all depend on maximum vibrational velocity and if maximum vibrational velocity increase, its corresponding natural-frequency of vibrations increase compare to forecast of linear system. Moreso, increasing tubes length, effects of maximum vibrational velocity on natural frequencies decrease as well. In addition, it also depicted that, quantity and variations of nonlinear natural-frequencies are evident in highly vibrational mode and two fixed end condition. Conclusively, results obtained in this studied are compared to order four Runge-Kuta numerical solutions and others result in literatures and there are all in accurate agreement as the results and can be compared with future investigation.

Shaba et al. (2021) studied nonlinear vibrational analysis of single-walled carbon nanotubes (SWCNTs) with the aid of longitudinal magnetic-field under Euler Bernoulli Beams and Eringen's non-local elasticity theories. Moreso, one parameter FEM together with Newmark time integration method was adopted to analyzed effects of nonlocal parameters of small-scale length, fluid velocity and magnetic fields permeability/strength on the deflections of the SWCNTs. Furthermore, MATLAB was adopted for parametric study of the parameters to obtained dynamic response of the systems under investigation and the results obtained shown that fluid velocity parameters affected mostly than the deflection of CNTs, nonlocal parameters and magnetic field permeability/strength.

A. Ghorbanpour Arani et al (2015) investigated flexural vibrational stability analysis of coupled double-walled visco-elastic carbon nanotubes conveying fluid under Timoshenko beams (TB) model theory and the couple system is resting under spring medium and Pasternak type-foundation. Moreso, Lenard-Jones model uses van der Waals (vdW) force between the inner-outer DWCNTs was considered. Furthermore, Hamilton's principle, small scale theories and application of 2D magnetic fields of higher order nonlinear governing equation of motions is obtained and the analytical solution of differential quadrature (DQ) was apply on the nonlinear PDEs of motions to obtained dynamic response of the system under study. Finally, effects of visco-elastic, magnetic fields with variable magnitude and surface stress on natural frequencies of the structures are confirmed under the investigation.

Ebrahimi and Nasirzadeh (2015) presented dynamic vibration analysis of single-walled carbon nanotube (SWCNT) under the following boundary conditions; simply, clamped and free. Furthermore, through variationally formulation and Hamilton's principle couple with Eringen's nonlocal elasticity and Timoshenko beams theories, the nonlinear governing partial differential equation of motions was developed. Moreso, the developed nonlinear governing equation was solve by partial differential transform method (DTM) to obtained dynamical responses of the systems and the obtain results was compared with other well-known methods and there are in good accuracy. Also, effects of simulation of transverse shear-deformation effect, slenderness ratio, boundary conditions and small-scale vibrational characteristic of single-walled carbon nanotubes was investigated respectively. Conclusively, the investigation revealed that, vibrational characteristic of single walled carbon nanotubes are believed to rely on nonlocal small-scale parameters as seen during the study.

M.M.S. Fakhrabadi et al. (2015) investigated effects of applying of nonlocal elasticity theories on electro-mechanical behavior of single-walled carbon nanotube (SWCNTs) under electrostatically actuations. Furthermore, influence of several dimension and ends condition on vibrational and dynamical instabilities analysis of single-walled carbon nanotube is considered. The obtained results revealed that applying nonlocal elasticity theory led to the higher pull-in voltage for the non-local models used for the single-walled carbon nanotube. Moreso, to achieved good agreeable result, application of non-classical theory such as non-local elasticity theories to examine mechanic and electro-mechanical behaviors of the nanostructures.

Zahra et al. (2021) analyzed nonlinear couple torsional radial vibrational analysis of SWCNTs with doublet mechanics (DM) principles to obtained two PDEs of governing nonlinear couple torsional radial vibrational for such CNTs. Furthermore, these PDEs of motions is reduced to more convenient duffing equation with the assistance of Galerkin decomposition method. Subsequently, the resulting duffing equation is solve using a semi-analytical approach of homotopic perturbation method (HPM) to obtained dynamic response such as complicated frequencies but due to coupling of the two vibrational modes. Again, dependent of end conditions, vibrational mode and carbon nanotube geometry on nonlinear couple torsional-radial vibrational characteristic of single walled carbon nanotube is considered. Also, ends support condition and maximum vibrational velocities have important effects on the nonlinear coupled torsional radial vibrational responses of SWCNTs. Subsequently, linear model, maximum vibrational velocity increases with corresponding increases in natural-frequencies of vibration. The obtained results was compared with fourth order Runge-Kuta numerical solutions and there are in accuracy agreement as the new obtain results can be use as benched mark for future investigations.

Xu et al. (2018) developed nonlinear model of single-walled carbon nanotube (SWCNTs) and study nonlinear dynamical characteristic of such single-walled carbon nanotubes which is subjected to random magnetic fields permeability/strength. Furthermore, Eringen's differential constitutive model considered effects of non-local microstructure and energy function method (EFM) is used to obtained the nonlinear dynamical response like natural frequency. Also, the drift and diffusion coefficients are authentic. Subsequently, semi-analytical solutions and numerical parametric investigation depicted that stochastical resonance happened when vary random magnetic fields intensities. Again, boundaries of safe basins have fractal characteristics and decreasing the areas of safe basins increasing corresponding intensities of the magnetic fields permeabilities.

2. FORMULATION OF GOVERNING EQUATION OF MOTION

In order to modelled nonlinear governing equations of motions for slightly curved single-walled carbon nanotubes, the slightly curved SWCNTs under the influence of stretching effect and resting on Winkler-Pasternak foundations in a magneto-thermal environment is shown in Fig. 1. With the aid of the following; Eringen's nonlocal elasticity theory, Euler-Bernoulli beam theory and Hamilton's principle.

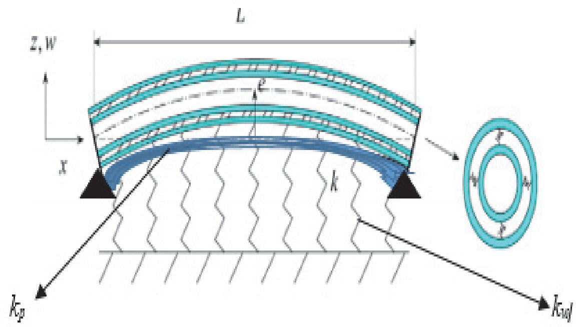


Figure 1: Slightly curved embedded CNTs on elastic foundation with different boundary conditions.

The nonlinear governing equation of motion for the CNTs as is developed as:

$$EI\frac{\partial^4 w}{\partial x^4}+m_c\frac{\partial^2 w}{\partial t^2}+k_1 w+k_3 w^3-\left(\frac{EA}{2L}\int_0^L\left(\frac{\partial Z_o}{\partial x}\frac{\partial w}{\partial x}+\left(\frac{\partial w}{\partial x}\right)^2\right)dx\right)\left(\frac{\partial^2 Z_o}{\partial x^2}+\frac{\partial^2 w}{\partial x^2}\right)-\left(EA\alpha_x T+\eta A H_x{}^2+k_p\right)\frac{\partial^2 w}{\partial x^2}$$
$$+(e_o a)^2\left(\begin{array}{l} m_c\dfrac{\partial^4 w}{\partial x^2\partial t^2}+k_1\dfrac{\partial^2 w}{\partial x^2}+6k_3 w\left(\dfrac{\partial w}{\partial x}\right)^2+3k_3 w^2\left(\dfrac{\partial^2 w}{\partial x^2}\right)\\ -\left(\dfrac{EA}{2L}\displaystyle\int_0^L\left(\dfrac{\partial Z_o}{\partial x}\dfrac{\partial w}{\partial x}+\left(\dfrac{\partial w}{\partial x}\right)^2\right)dx\right)\left(\dfrac{\partial^4 Z_o}{\partial x^4}+\dfrac{\partial^4 w}{\partial x^4}\right)\\ -\left(EA\alpha_x T+\eta A H_x{}^2+k_p\right)\dfrac{\partial^4 w}{\partial x^4}\end{array}\right)=F_e+F_c+(e_o a)^2\frac{\partial^2}{\partial x^2}\left[F_e+F_c\right] \quad (1)$$

F_e is electrostatic force per unit length due to voltage V

$$F_e=\frac{1}{2}\frac{b\varepsilon V^2}{(d_o-w)^2}\left(1+f\left(\frac{d_0-w}{b}\right)\right) \quad (2)$$

F_c is the Casimir intermolecular force per unit length

$$F_c = \frac{\pi^3 \bar{h} c b}{240\left(d_o - w\right)^4} \tag{3}$$

Substituting equations (2) and (3) into equation (1), the nonlinear dynamical behaviour of the electrostatic actuated carbon nanotubes under the impacts of Casimir forces becomes:

$$\begin{aligned}
& EI\frac{\partial^4 w}{\partial x^4} + m\frac{\partial^2 w}{\partial t^2} - \left(\frac{EA}{2L}\int_0^L \left(\frac{\partial Z_o}{\partial x}\frac{\partial w}{\partial x} + \left(\frac{\partial w}{\partial x}\right)^2\right) dx\right)\left(\frac{\partial^2 Z_o}{\partial x^2} + \frac{\partial^2 w}{\partial x^2}\right) - \begin{pmatrix} EA\alpha_x T \\ +\eta A H_x^{\ 2} + k_p \end{pmatrix}\frac{\partial^2 w}{\partial x^2} \\
& -\left(e_o a\right)^2 \left(\begin{aligned} & m\frac{\partial^4 w}{\partial x^2 \partial t^2} - \left(\frac{EA}{2L}\int_0^L \left(\frac{\partial Z_o}{\partial x}\frac{\partial w}{\partial x} + \left(\frac{\partial w}{\partial x}\right)^2\right) dx\right)\left(\frac{\partial^4 Z_o}{\partial x^4} + \frac{\partial^4 w}{\partial x^4}\right) - \begin{pmatrix} EA\alpha_x T + \\ \eta A H_x^{\ 2} + k_p \end{pmatrix}\frac{\partial^4 w}{\partial x^4} \\ & + \left(\frac{\partial^2 w}{\partial x^2} - \frac{\partial^2 w}{\partial x^2}\right) \end{aligned} \right) \\
& = \frac{1}{2}\frac{b\varepsilon V^2}{\left(d-w\right)^2}\left(1 + f\left(\frac{d-w}{b}\right)\right) + \frac{\pi^3 \bar{h} c b}{240\left(d-w\right)^4} + \left(e_o a\right)^2 \left[\begin{aligned} & \frac{b\varepsilon V^2}{\left(d-w\right)^4}\left(3 + f\left(\frac{d-w}{b}\right)\right) \\ & + \frac{\pi^3 \bar{h} c b}{20\left(d-w\right)^6} \end{aligned} \right]
\end{aligned} \tag{4}$$

$k_w w = k_1 w + k_3 w^3$, where, $w(x,t)$ is the bending deflection of the tube, t is the time coordinate, EI is the bending rigidity, m, is the mass of tube per unit length and Z_o is the initial curvature of the tube. $EA\alpha_x T$ denotes the constant axial force due to thermal effects where, A is the cross-sectional area of the tube, α_x is the coefficient of thermal expansivity and T is the change in temperature. $\eta A H_x^{\ 2}$ is the magnetic force per unit length due to Lorentz force exerted on the tube in z-direction and, the term η is the magnetic field permeability and H_x is the magnetic field strength.

However, this study presents the analysis and dynamical response of a SWCNTs subjected to displacements boundary conditions as shown below:

For Pinned-Pinned supported (P-P) nanotube,

$$w(0,t) = 0, \quad \frac{\partial^2 w(0,t)}{\partial^2 x} = 0, \quad w(L,t) = 0, \quad \frac{\partial^2 w(L,t))}{\partial^2 x} = 0 \tag{5}$$

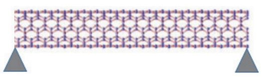
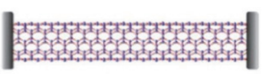
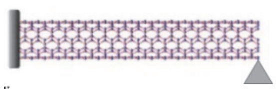
For Fixed-Fixed supported (F-F) nanotube,

$$w(0,t) = 0, \quad \frac{\partial w(0,t)}{\partial x} = 0, \quad w((L,t) = 0, \quad \frac{\partial w(L,t)}{\partial x} = 0 \tag{6}$$

For a Fixed-Pinned supported (F-P) nanotube,

$$w(0,t) = 0, \quad \frac{\partial w(0,t)}{\partial x} = 0, \quad w(L,t) = 0, \frac{\partial^2 w((L,t)}{\partial^2 x} = 0 \tag{7}$$

Table 1: The basic functions corresponding to the above boundary conditions

Cases	**Mode shape, ϕ(x)**	**Value of β**
1. Simply support 	$sin\left(\frac{\beta x}{L}\right)$	π
2. Clamped-Clamped support 	$\left(\cosh\left(\frac{\beta x}{L}\right) - \cos\left(\frac{\beta x}{L}\right)\right) - \left(\frac{\sinh\beta + \sin\beta}{\cosh\beta - \cos\beta}\right)\left(\sinh\left(\frac{\beta x}{L}\right) - \sin\left(\frac{\beta x}{L}\right)\right)$	4.730041
3. Clamped-Simply support 	$\left(\cosh\left(\frac{\beta x}{L}\right) - \cos\left(\frac{\beta x}{L}\right)\right) - \left(\frac{\cosh\beta - \cos\beta}{\sinh\beta - \sin\beta}\right)\left(\sinh\left(\frac{\beta x}{L}\right) - \sin\left(\frac{\beta x}{L}\right)\right)$	3.926602

2.1. Converting the governing equation from pde into ode using galerkin decomposition method

For convenience, equation (4) is rearranged into dimensionless form using some dimensionless parameters. These dimensionless variables are:

$$X=\frac{x}{L},\quad W=\frac{w}{d_0},\quad K_1=\frac{k_1L^4}{EI},\quad \tau=t\frac{h}{L^2}\sqrt{\frac{E}{12\rho}},\quad H=\frac{\eta A{H_x}^2L^2}{EI}$$

$$\lambda_3=\frac{L^4}{EId_0}\left(\frac{AbL^4}{6EI\pi d_0^4}+\frac{bf\varepsilon V^2\left(e_0a\right)^2}{d_0^3}\right),\quad \lambda_4=\frac{L^4}{EId_0}\left(\frac{\pi^2hcbL^4}{240EId_0^5}+\frac{\left(e_0a\right)^2\left(3b\varepsilon V^2+\frac{Ab}{\pi}\right)}{d_0^4}\right) \tag{8}$$

$$\lambda_6=\frac{\pi^2hcbL^4}{20EId_0^7},\quad K_3=\frac{k_3L^4d_0^2}{EI},\quad \alpha_1=\frac{16d_0^2}{h_2},\quad K_p=\frac{k_pL^2}{EI},\quad \theta=\frac{EA\alpha_xTL^2}{EI}$$

Substituting Eq. (8) into (4), the dimensionless equation of motion becomes as illustrated in Eq. (9) below:

$$\frac{\partial^4W}{\partial X^4}+\frac{\partial^2W}{\partial\tau^2}+K_1W+K_3W^3-\left(\alpha_1\int_0^1\left(\frac{\partial Z_o}{\partial X}\frac{\partial W}{\partial X}+\left(\frac{\partial W}{\partial X}\right)^2\right)dX\right)\left(\frac{\partial^2Z_o}{\partial X^2}+\frac{\partial^2W}{\partial X^2}\right)-\left(\theta+H+K_p\right)\frac{\partial^2W}{\partial X^2}$$
$$+\left(e_oa\right)^2\left(\begin{array}{l}\alpha_2\frac{\partial^4W}{\partial X^2\partial\tau^2}+K_1\frac{\partial^2W}{\partial X^2}+6K_3W\left(\frac{\partial W}{\partial X}\right)^2+3K_3W^2\left(\frac{\partial^2W}{\partial X^2}\right)\\-\left(\alpha_1\int_0^1\left(\frac{\partial Z_o}{\partial X}\frac{\partial W}{\partial X}+\left(\frac{\partial W}{\partial X}\right)^2\right)dX\right)\left(\frac{\partial^4Z_o}{\partial X^4}+\frac{\partial^4W}{\partial X^4}\right)\\-\left(\theta+H+K_p\right)\frac{\partial^4W}{\partial W^4}\end{array}\right)=\frac{\lambda_1}{(1-W)}+\frac{\lambda_2}{(1-W)^2}+\frac{\lambda_3}{(1-W)^3}+\frac{\lambda_4}{(1-W)^4}+\frac{\lambda_6}{(1-W)^6} \tag{9}$$

By expansion,

$$\lambda_1/(1-W)=\lambda_1\left(1+W+W^2+W^3+W^4+W^5+W^6+O\left(W^7\right)\right) \tag{10}$$

$$\lambda_2/(1-W)^2=\lambda_2\left(1+2W+3W^2+4W^3+5W^4+6W^5+7W^6+O\left(W^7\right)\right) \tag{11}$$

$$\lambda_3/(1-W)^3=\lambda_3\left(1+3W+6W^2+10W^3+15W^4+21W^5+28W^6+O\left(W^7\right)\right) \tag{12}$$

$$\lambda_4/(1-W)^4=\lambda_4\left(1+4W+10W^2+20W^3+35W^4+56W^5+84W^6+O\left(W^7\right)\right) \tag{13}$$

$$\lambda_6/(1-W)^6=\lambda_6\left(1+6W+21W^2+56W^3+126W^4+252W^5+462W^6+O\left(W^7\right)\right) \tag{14}$$

Substituting Eq. (10) to (14) into Eq. (9) and grouping like terms, we have:

$$\frac{\partial^4W}{\partial X^4}+\frac{\partial^2W}{\partial\tau^2}+K_1W+K_3W^3-\left(\alpha_1\int_0^1\left(\frac{\partial Z_o}{\partial X}\frac{\partial W}{\partial X}+\left(\frac{\partial W}{\partial X}\right)^2\right)dX\right)\left(\frac{\partial^2Z_o}{\partial X^2}+\frac{\partial^2W}{\partial X^2}\right)-\left(\theta+H+K_p\right)\frac{\partial^2W}{\partial X^2}$$
$$+\left(e_oa\right)^2\left(\begin{array}{l}\alpha_2\frac{\partial^4W}{\partial X^2\partial\tau^2}+K_1\frac{\partial^2W}{\partial X^2}+6K_3W\left(\frac{\partial W}{\partial X}\right)^2+3K_3W^2\left(\frac{\partial^2W}{\partial X^2}\right)\\-\left(\alpha_1\int_0^1\left(\frac{\partial Z_o}{\partial X}\frac{\partial W}{\partial X}+\left(\frac{\partial W}{\partial X}\right)^2\right)dX\right)\left(\frac{\partial^4Z_o}{\partial X^4}+\frac{\partial^4W}{\partial X^4}\right)\\-\left(\theta+H+K_p\right)\frac{\partial^4W}{\partial W^4}\end{array}\right)$$
$$=\left(\begin{array}{l}\left(\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_6\right)+\left(\lambda_1+2\lambda_2+3\lambda_3+4\lambda_4+6\lambda_6\right)W+\left(\begin{array}{l}\lambda_1+3\lambda_2+6\lambda_3\\+10\lambda_4+21\lambda_6\end{array}\right)W^2\\+\left(\lambda_1+4\lambda_2+10\lambda_3+20\lambda_4+56\lambda_6\right)W^3+\left(\lambda_1+5\lambda_2+15\lambda_3+35\lambda_4+126\lambda_6\right)W^4\\+\left(\lambda_1+6\lambda_2+21\lambda_3+56\lambda_4+252\lambda_6\right)W^5+\left(\lambda_1+7\lambda_2+28\lambda_3+84\lambda_4+462\lambda_6\right)W^6\end{array}\right)$$

To convert Eq. (15) into Ordinary Differential Equation, the Galerkin decomposition method is employed. This procedure allows the deflection of the SWCNTs to be represented as a product of two independent functions as shown;

$$W(X,\tau)=U(\tau)\,\phi(X) \tag{16}$$

Where ϕ(x) as expressed in table 3.1, is a function selected to satisfy the boundary conditions. Recall that the Galerkin one parameter at a time transform is defined as;

$$\int_0^1 R(X,\tau)\phi(X)\,dX \tag{17}$$

Where $R(X,\tau)$ is the nonlinear equation. Applying Eq. (17) to the governing equations, the decomposition equation becomes;

$$\int_0^1 \left(\begin{array}{l} \dfrac{\partial^4 W}{\partial X^4}+\dfrac{\partial^2 W}{\partial \tau^2}+K_1W+K_3W^3-\left(\alpha_1\displaystyle\int_0^1\left(\dfrac{\partial Z_o}{\partial X}\dfrac{\partial W}{\partial X}+\left(\dfrac{\partial W}{\partial X}\right)^2\right)dX\right)\left(\dfrac{\partial^2 Z_o}{\partial X^2}+\dfrac{\partial^2 W}{\partial X^2}\right) \\ -\left(\theta+H+K_p\right)\dfrac{\partial^2 W}{\partial X^2}+(e_oa)^2\left(\begin{array}{l} \alpha_2\dfrac{\partial^4 W}{\partial X^2\partial\tau^2}+K_1\dfrac{\partial^2 W}{\partial X^2}+6K_3W\left(\dfrac{\partial W}{\partial X}\right)^2+3K_3W^2\left(\dfrac{\partial^2 W}{\partial X^2}\right) \\ -\left(\alpha_1\displaystyle\int_0^1\left(\dfrac{\partial Z_o}{\partial X}\dfrac{\partial W}{\partial X}+\left(\dfrac{\partial W}{\partial X}\right)^2\right)dX\right)\left(\dfrac{\partial^4 Z_o}{\partial X^4}+\dfrac{\partial^4 W}{\partial X^4}\right) \\ -\left(\theta+H+K_p\right)\dfrac{\partial^4 W}{\partial W^4} \end{array}\right) \\ =\left(\begin{array}{l} (\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_6)+(\lambda_1+2\lambda_2+3\lambda_3+4\lambda_4+6\lambda_6)W+\left(\begin{array}{l}\lambda_1+3\lambda_2+6\lambda_3\\+10\lambda_4+21\lambda_6\end{array}\right)W^2 \\ +(\lambda_1+4\lambda_2+10\lambda_3+20\lambda_4+56\lambda_6)W^3+(\lambda_1+5\lambda_2+15\lambda_3+35\lambda_4+126\lambda_6)W^4 \\ +(\lambda_1+6\lambda_2+21\lambda_3+56\lambda_4+252\lambda_6)W^5+(\lambda_1+7\lambda_2+28\lambda_3+84\lambda_4+462\lambda_6)W^6 \end{array}\right) \end{array}\right)\phi(X)\,dX \tag{18}$$

Performing the integration and grouping using the temporal term as coefficient, the Duffing equation for the SWCNT becomes:

$$M\ddot{U}+G\dot{U}+\sum_{\zeta=0}^{6}C_\zeta U^\zeta=0 \tag{19}$$

Which in expanded for gives;

$$M\ddot{U}+G\dot{U}+C_0+C_1U+C_2U^2+C_3U^3+C_4U^4+C_5U^5+C_6U^6=0 \tag{20}$$

Subject to $U=a,\quad \dot{U}=0$

Where,

$$M=\int_0^1\left[\phi(X)+\alpha_2\frac{d^2\phi(X)}{\partial X^2}\right]\phi(X)\,dX$$

$$C_0=-\int_0^1[\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_6]\phi(X)\,dX$$

$$C_1=\int_0^1\left[\begin{array}{l} \dfrac{d^4\phi(X)}{dX^4}+K_1\phi(X)-(\alpha_1N)\left(\dfrac{d^2\phi(X)}{dX^2}\right)-\left(\theta+H+K_p\right)\dfrac{d^2\phi(X)}{dX^2} \\ +(e_oa)^2\left(K_1\dfrac{d^2\phi(X)}{dX^2}-(\alpha_1N)\left(\dfrac{d^4\phi(X)}{dX^4}\right)-\left(\theta+H+K_p\right)\dfrac{d^4\phi(X)}{dX^4}\right) \\ -(\lambda_1+2\lambda_2+3\lambda_3+4\lambda_4+6\lambda_6) \end{array}\right]\phi(X)\,dX$$

$$C_2=-\int_0^1[\lambda_1+3\lambda_2+6\lambda_3+10\lambda_4+21\lambda_6]\phi(X)\,dX$$

$$C_3 = \int_0^1 \begin{bmatrix} K_3\phi^3(X) + (e_o a)^2 \left(6K_3\phi(X)\left(\frac{d\phi(X)}{dX}\right)^2 + 3K_3\phi(X)^2\left(\frac{d^2\phi(X)}{dX^2}\right)\right) \\ -(\lambda_1 + 4\lambda_2 + 10\lambda_3 + 20\lambda_4 + 56\lambda_6) \end{bmatrix} \phi(X)dX$$

$$C_4 = -\int_0^1 [\lambda_1 + 5\lambda_2 + 15\lambda_3 + 35\lambda_4 + 126\lambda_6]\phi^5(X)dX$$

$$C_5 = -\int_0^1 [\lambda_1 + 6\lambda_2 + 21\lambda_3 + 56\lambda_4 + 252\lambda_6]\phi^6(X)dX$$

$$C_6 = -\int_0^1 [\lambda_1 + 7\lambda_2 + 28\lambda_3 + 84\lambda_4 + 462\lambda_6]\phi^7(X)dX$$

$$G = 0$$

3. SOLUTION METHODOLOGY

The homotopy perturbation method was introduced by Ji Huan HE in 1998. This method become more popular and acceptability as an elegant tool in the hand of researchers due to its simplicity in nature and give rises to highly effective solution of complicated nonlinear problems in sevral diverse areas of science and technological based knowledge**.** The technique was based on homotopic, which is the significant part of topology. Interesting property of homotopy, is that one can transformed any nonlinear problems into an infinite number of linear problems, no matter whether or not there exists a smaller or larger parameter. To illustrate this general procedure, let us consider a general nonlinear partial differential equation of the form;

In this section we have illustrated the basic ideas behind HPM is to solve nonlinear equations. Let us considered the following nonlinear differential equations of the form;

$$\mathrm{A}(u) - f(r) = 0, r \in \Omega \tag{21}$$

Subject to boundary condition:

$$\mathrm{B}\left(u, \frac{\partial u}{\partial n}\right) = 0, \quad r \in \Gamma \tag{22}$$

where A is a general differential operator, B a boundary operator, f(r) a known analytical function and Γ is the boundary of the domain Ω. In general, one can divide the operator A into two parts: linear and non-linear. That means

$$A = L + N \tag{23}$$

where *L* is linear and *N* is non-linear.

Hence, equation (21) can now be rewritten as

$$L(u) + \mathrm{N}(u) - f(r) = 0, \quad r \in \Omega \tag{24}$$

By the homotopy technique, one can construct a homotopy in the following way $v(r,p): \Omega\, x\, [0,1] \rightarrow R$ which satisfies

$$\mathrm{H}(v,p) = (1-p)\left[L(v) - L(u_0)\right] + p\left[\mathrm{A}(v) - f(r)\right] = 0, \quad p \in [0,1], \quad r \in \Omega \tag{25}$$

or

$$\mathrm{H}(v,p) = L(v) - L(u_0) + p\left[\mathrm{N}(v) - f(r)\right] = 0 \tag{26}$$

where $p \in [0,1]$ is an embedding parameter, u_0 is an initial approximation which satisfies the boundary conditions.

$$H(v,0) = L(v) - L(u_0) = 0 \tag{27}$$

$$H(v,1) = \mathrm{A}(v) - f(r) = 0 \tag{28}$$

The changing process of *p* from zero to unity is just that of *v(r,p)* from $u_0(r)$ to *u(r)*. In topology, this is called deformation and $L(v)$-$L(u_0)$ and *A(v)-f(r)* are called homotopy. According to the HPM, we can first use the embedding parameter *p* as a "small parameter" and assume that the solution be written as a power series in *p*

$$v = v_o + pv_1 + p^2v_2 + \dots \tag{29}$$

Setting $p = 1$ result in the approximate solution:

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + \dots \tag{30}$$

The combination of the perturbation and homotopy methods are called the homotopy perturbation method (HPM), which has eliminated the limitations of the traditional perturbation methods. On the other hand, this method has full merit of the traditional perturbation method.

Applying Homotopy Perturbation Method on Duffing Equation

$$\mathrm{M}\ddot{U}+C_0+C_1U+C_2U^2+C_3U^3+C_4U^4+C_5U^5+C_6U^6=0 \tag{31}$$

Divide through by M

$$\ddot{U}+\frac{C_0}{\mathrm{M}}+\frac{C_1}{\mathrm{M}}U+\frac{C_2}{\mathrm{M}}U^2+\frac{C_3}{\mathrm{M}}U^3+\frac{C_4}{\mathrm{M}}U^4+\frac{C_5}{\mathrm{M}}U^5+\frac{C_6}{\mathrm{M}}U^6=0 \tag{32}$$

$$\text{Let, } \beta_0=\frac{C_0}{\mathrm{M}},\beta_1=\frac{C_1}{\mathrm{M}},\beta_2=\frac{C_2}{\mathrm{M}},\beta_3=\frac{C_3}{\mathrm{M}},\beta_4=\frac{C_4}{\mathrm{M}},\beta_5=\frac{C_5}{\mathrm{M}},\beta_6=\frac{C_6}{\mathrm{M}}, \tag{33}$$

Substitute equation (33) into equation (32) gives;

$$\ddot{U}+\beta_0+\beta_1U+\beta_2U^2+\beta_3U^3+\beta_4U^4+\beta_5U^5+\beta_6U^6=0 \tag{34}$$

Apply, HPM on equation (34) becomes:

$$\mathrm{H}(v,p)=(1-p)\left[\ddot{U}+\beta_1U\right]+p\left[\ddot{U}+\beta_0+\beta_1U+\beta_2U^2+\beta_3U^3+\beta_4U^4+\beta_5U^5+\beta_6U^6\right]=0,\quad p\in[0,1],\quad r\in\Omega \tag{35}$$

Perturbating the parameters become (36) & (37) are obtain as follows;

$$\left.\begin{aligned}\ddot{U}&=U_0''+pU_1''+p^2U_2''+\dots\\ \dot{U}&=U_0'+pU_1'+p^2U_2'+\dots\\ U&=U_0+pU_1+p^2U_2+\dots\end{aligned}\right] \tag{36}$$

And

$$\beta_1=\omega_0^2+p\omega_1^2+p^2\omega_2^2+\dots \tag{37}$$

Substitute equation (36) & (37) into equation (35)

$$\begin{aligned}&(1-p)\left[U_0''+pU_1''+p^2U_2''+\dots+\left(\omega_0^2+p\omega_1^2+p^2\omega_2^2+\dots\right)\left(U_0+pU_1+p^2U_2+\dots\right)\right]+\\ &p\begin{bmatrix}U_0''+pU_1''+p^2U_2''+\dots+\left(\omega_0^2+p\omega_1^2+p^2\omega_2^2+\dots\right)\left(U_0+pU_1+p^2U_2+\dots\right)+\\ \beta_2\left(U_0+pU_1+p^2U_2+\dots\right)^2+\beta_3\left(U_0+pU_1+p^2U_2+\dots\right)^3+\\ \beta_4\left(U_0+pU_1+p^2U_2+\dots\right)^4+\beta_5\left(U_0+pU_1+p^2U_2+\dots\right)^5+\\ \beta_6\left(U_0+pU_1+p^2U_2+\dots\right)^6+\beta_0\end{bmatrix}=0\end{aligned} \tag{38}$$

Expanding equation (38) and collecting all terms with the same order of p together, the following

resulting equations appears in form of polynomial in power of p are obtain. Where, u_i and ω_i^2, $i=0,1,2,\dots$ and equated same to zero as shown below;

$$\mathrm{P}^0: U_0''+\omega_0^2U_0=0 \text{ boundaryconditionas } U_0(0)=\mathrm{A} \text{ and } \dot{U}_0(0)=0 \tag{39}$$

$$\mathrm{P}^1: U_1''+\omega_0^2U_1+\omega_1^2U_0+\beta_2U_0^2+\beta_2U_0^3+\beta_2U_0^4+\beta_2U_0^5+\beta_2U_0^6+\beta_0=0$$

Boundary condition: $U_1(0)=0$ and $\dot{U}_1(0)=0$ (40)

Using initial conditions in equation (40) which give the solution as;

$$U_0=A\cos\omega_0t \tag{41}$$

Substituting the value of equation (41) into equation (34) yields;

$$\begin{pmatrix}U_1''+\omega_0^2U_1+\omega_1^2\left(A\cos\omega_0t\right)+\beta_2(A\cos\omega_0t)^2+\beta_3(A\cos\omega_0t)^3+\\ \beta_4(A\cos\omega_0t)^4+\beta_5(A\cos\omega_0t)^5+\beta_6(A\cos\omega_0t)^6+\beta_0\end{pmatrix}=0 \tag{42}$$

Performing trigonometric identities function on equation (41) gives;

$$\left(\begin{array}{l} \ddot{U}(t)+\omega_0{}^2U(t)_1+\omega_1^2A\cos(\omega t)+\frac{\beta_2A^2}{2}\cos(2\omega t)+\frac{\beta_2A^2}{2}+\frac{\beta_3A^3}{4}\cos(3\omega t)+ \\ \frac{3\beta_3A^3}{4}\cos(\omega t)+\frac{\beta_4A^4}{8}\cos(4\omega t)+\frac{\beta_4A^4}{2}\cos(2\omega t)+ \\ \frac{3\beta_4A^4}{8}+\frac{\beta_5A^5}{16}\cos(5\omega t)+\frac{5\beta_5A^5\cos(3\omega t)}{16}+\frac{5\beta_5A^5}{8}\cos(\omega t)+ \\ \frac{\beta_6A^6}{32}\cos(6\omega t)+\frac{3\beta_6A^6}{16}\cos(4\omega t)+\frac{15\beta_6A^6\cos(2\omega t)}{32}+\frac{5\beta_6A^6}{16}+\beta_0=0 \end{array}\right) \tag{43}$$

Neglecting secular terms, equation (43) becomes;

$$\left(\omega_1^2A+\frac{3\beta_3A^3}{4}+\frac{5\beta_5A^5}{8}\right)\cos(\omega t)=0 \tag{44}$$

And Nonlinear frequency ω_1 is obtain as

$$\omega_1=\sqrt{-\left(\frac{3\beta_3\mathrm{A}^2}{4}+\frac{5\beta_5\mathrm{A}^4}{8}\right)} \tag{45}$$

For the Model Stability Analysis

From equation (37), setting $p=1,\ \beta_1=\omega_0^2+\omega_1^2+\ldots=0$ (46)

Substitute ω_1^2 into equation (46), gives:

$$\beta_1=\omega_0^2-\frac{3\beta_3\mathrm{A}^2}{4}-\frac{5\beta_5\mathrm{A}^4}{8} \tag{47}$$

$$\omega_0=\sqrt{\beta_1+\frac{3\beta_3\mathrm{A}^2}{4}+-\frac{5\beta_5\mathrm{A}^4}{8}} \tag{48}$$

Natural frequency is obtained as;

$$\omega_n=\sqrt{\frac{C_1}{M}}=\sqrt{\beta_1} \tag{49}$$

Therefore, frequency ratio is obtained as;

$$\frac{\omega_0}{\omega_n}=\frac{\sqrt{\beta_1+\frac{3\beta_3\mathrm{A}^2}{4}+-\frac{5\beta_5\mathrm{A}^4}{8}}}{\sqrt{\beta_1}}=\sqrt{1+\frac{1}{\beta_1}\left(\frac{3\beta_3\mathrm{A}^2}{4}+-\frac{5\beta_5\mathrm{A}^4}{8}\right)} \tag{50}$$

Solving equation (42) and neglecting secular terms with the boundary conditions

$$u_1(t)=\frac{1}{1680\omega_0^2}\left(\begin{array}{l} 48A^6\left(\cos(\omega_0t)\right)^6\beta_6+96\beta_6A^6\left(\cos(\omega_0t)\right)^4 \\ +350A^5\left(\cos(\omega_0t)\right)^5\beta_5+384A^6\beta_6\left(\cos(\omega_0t)\right)^2 \\ -175A^5\left(\cos(\omega_0t)\right)^3\beta_5+112A^4\left(\cos(\omega_0t)\right)^4\beta_4 \\ +240A^6\cos(\omega_0t)\beta_6-768\beta_6A^6-175\beta_5A^5\cos(\omega_0t) \\ +448A^4\beta_4\left(\cos(\omega_0t)\right)^2+630A^3\left(\cos(\omega_0t)\right)^3\beta_3 \\ +336A^4\cos(\omega_0t)\beta_4-896\beta_4A^4-630\beta_3A^3\cos(\omega_0t) \\ +560A^2\left(\cos(\omega_0t)\right)^2\beta_2+560A^2\cos(\omega_0t)\beta_2 \\ -1120\beta_2A^2+1680\cos(\omega_0t)\beta_0-1680\beta_0 \end{array}\right) \tag{51}$$

Obtaining the series solutions as follows;

$$u(t)=u_0(t)+u_1(t)+\ldots \quad (52)$$

$$u(t)=A\cos(\omega_0 t)+\frac{1}{1680\omega_0^2}\begin{pmatrix} 48A^6\left(\cos(\omega_0 t)\right)^6\beta_6+96\beta_6A^6\left(\cos(\omega_0 t)\right)^4 \\ +350A^5\left(\cos(\omega_0 t)\right)^5\beta_5+384A^6\beta_6\left(\cos(\omega_0 t)\right)^2 \\ -175A^5\left(\cos(\omega_0 t)\right)^3\beta_5+112A^4\left(\cos(\omega_0 t)\right)^4\beta_4 \\ +240A^6\cos(\omega_0 t)\beta_6-768\beta_6A^6-175\beta_5A^5\cos(\omega_0 t) \\ +448A^4\beta_4\left(\cos(\omega_0 t)\right)^2+630A^3\left(\cos(\omega_0 t)\right)^3\beta_3 \\ +336A^4\cos(\omega_0 t)\beta_4-896\beta_4A^4-630\beta_3A^3\cos(\omega_0 t) \\ +560A^2\left(\cos(\omega_0 t)\right)^2\beta_2+560A^2\cos(\omega_0 t)\beta_2 \\ -1120\beta_2A^2+1680\cos(\omega_0 t)\beta_0-1680\beta_0 \end{pmatrix}+\ldots \quad (54)$$

Deflection is obtained for different carbon nanotubes ends conditions;

$$w(x,t)=u(t)*\phi(x) \quad (55)$$

For Simple-Simple supports

$$w(x,t)=\left[A\cos(\omega_0 t)+\frac{1}{1680\omega_0^2}\begin{pmatrix} 48A^6\left(\cos(\omega_0 t)\right)^6\beta_6+96\beta_6A^6\left(\cos(\omega_0 t)\right)^4 \\ +350A^5\left(\cos(\omega_0 t)\right)^5\beta_5+384A^6\beta_6\left(\cos(\omega_0 t)\right)^2 \\ -175A^5\left(\cos(\omega_0 t)\right)^3\beta_5+112A^4\left(\cos(\omega_0 t)\right)^4\beta_4 \\ +240A^6\cos(\omega_0 t)\beta_6-768\beta_6A^6-175\beta_5A^5\cos(\omega_0 t) \\ +448A^4\beta_4\left(\cos(\omega_0 t)\right)^2+630A^3\left(\cos(\omega_0 t)\right)^3\beta_3 \\ +336A^4\cos(\omega_0 t)\beta_4-896\beta_4A^4-630\beta_3A^3\cos(\omega_0 t) \\ +560A^2\left(\cos(\omega_0 t)\right)^2\beta_2+560A^2\cos(\omega_0 t)\beta_2 \\ -1120\beta_2A^2+1680\cos(\omega_0 t)\beta_0-1680\beta_0 \end{pmatrix}\right]*\sin\left(\frac{\beta x}{l}\right) \quad (56)$$

For Clamped-Clamped supports

$$w(x,t)=\left[\begin{matrix} A\cos(\omega_0 t) \\ +\frac{1}{1680\omega_0^2}\begin{pmatrix} 48A^6\left(\cos(\omega_0 t)\right)^6\beta_6+96\beta_6A^6\left(\cos(\omega_0 t)\right)^4 \\ +350A^5\left(\cos(\omega_0 t)\right)^5\beta_5+384A^6\beta_6\left(\cos(\omega_0 t)\right)^2 \\ -175A^5\left(\cos(\omega_0 t)\right)^3\beta_5+112A^4\left(\cos(\omega_0 t)\right)^4\beta_4 \\ +240A^6\cos(\omega_0 t)\beta_6-768\beta_6A^6-175\beta_5A^5\cos(\omega_0 t) \\ +448A^4\beta_4\left(\cos(\omega_0 t)\right)^2+630A^3\left(\cos(\omega_0 t)\right)^3\beta_3 \\ +336A^4\cos(\omega_0 t)\beta_4-896\beta_4A^4-630\beta_3A^3\cos(\omega_0 t) \\ +560A^2\left(\cos(\omega_0 t)\right)^2\beta_2+560A^2\cos(\omega_0 t)\beta_2 \\ -1120\beta_2A^2+1680\cos(\omega_0 t)\beta_0-1680\beta_0 \end{pmatrix}\end{matrix}\right]*\begin{pmatrix} \cosh\left(\frac{\beta x}{l}\right)-\cos\left(\frac{\beta x}{l}\right) \\ -\frac{\sinh(\beta)+\sin(\beta)}{\cosh(\beta)-\cos(\beta)} \\ \left(\sinh\left(\frac{\beta x}{l}\right)-\sin\left(\frac{\beta x}{l}\right)\right) \end{pmatrix} \quad (56)$$

For Clamped-Simple supports

$$w(x,t)=\left[\begin{array}{l} A\cos(\omega_0 t) \\ +\dfrac{1}{1680\omega_0^2}\left(\begin{array}{l} 48A^6\left(\cos(\omega_0 t)\right)^6\beta_6+96\beta_6A^6\left(\cos(\omega_0 t)\right)^4 \\ +350A^5\left(\cos(\omega_0 t)\right)^5\beta_5+384A^6\beta_6\left(\cos(\omega_0 t)\right)^2 \\ -175A^5\left(\cos(\omega_0 t)\right)^3\beta_5+112A^4\left(\cos(\omega_0 t)\right)^4\beta_4 \\ +240A^6\cos(\omega_0 t)\beta_6-768\beta_6A^6-175\beta_5A^5\cos(\omega_0 t) \\ +448A^4\beta_4\left(\cos(\omega_0 t)\right)^2+630A^3\left(\cos(\omega_0 t)\right)^3\beta_3 \\ +336A^4\cos(\omega_0 t)\beta_4-896\beta_4A^4-630\beta_3A^3\cos(\omega_0 t) \\ +560A^2\left(\cos(\omega_0 t)\right)^2\beta_2+560A^2\cos(\omega_0 t)\beta_2 \\ -1120\beta_2A^2+1680\cos(\omega_0 t)\beta_0-1680\beta_0 \end{array}\right) \end{array}\right]*\left(\begin{array}{l} \cosh\left(\dfrac{\beta x}{l}\right)-\cos\left(\dfrac{\beta x}{l}\right) \\ -\dfrac{\cosh(\beta)-\cos(\beta)}{\sinh(\beta)-\sin(\beta)} \\ \left(\sinh\left(\dfrac{\beta x}{l}\right)-\sin\left(\dfrac{\beta x}{l}\right)\right) \end{array}\right) \quad (57)$$

Figure 1, 2, and 3 shows, first-fifth normalized mode shapes of the beam for the nanotubes with pinned-pinned, fixed-fixed and fixed-pinned supports. These figures depict the deflections of the beams along the beams' span at five different buckled mode shapes. The natural frequencies show where the maximum vibration occurs. The mode shapes show the deformation that the component would show when vibrating at the natural frequency. This mean that the mode shapes tell us how the structure tends to deform at the specific natural frequencies. Therefore, such analyses of the mode shapes and natural frequencies are very significant as they reveal the important property of the mechanical system and the system frequencies that is vulnerable or susceptible to vibration. They also depict when the natural frequencies coincide with the resonant frequencies of the system.

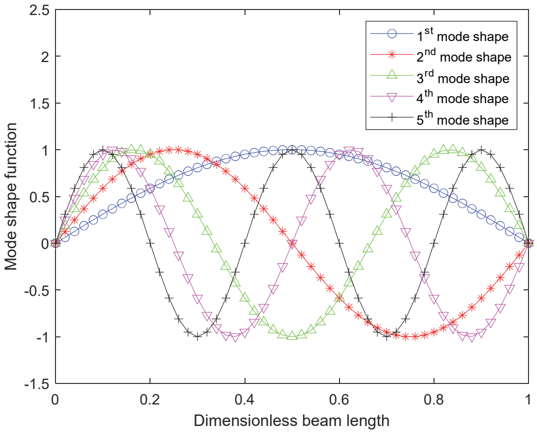


Figure 1: First-fifth normalized mode shapes of the beam for the nanotubes with pinned-pinned supports

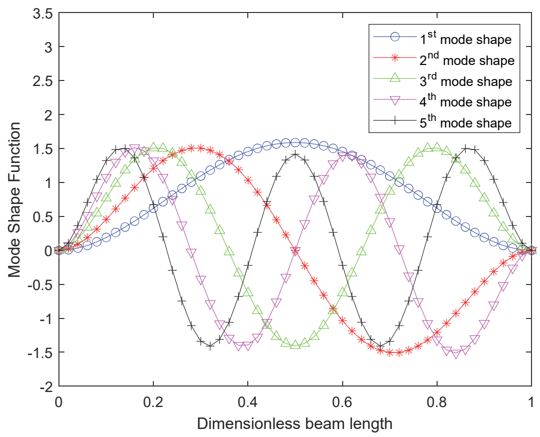


Figure 2: First-fifth normalized mode shapes of the beam for the nanotubes with fixed-fixed supports

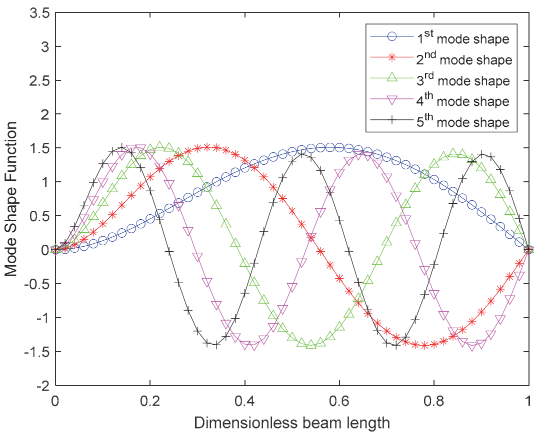


Figure 3: First-fifth normalized mode shapes of the beam for the nanotubes with fixed-pinned supports

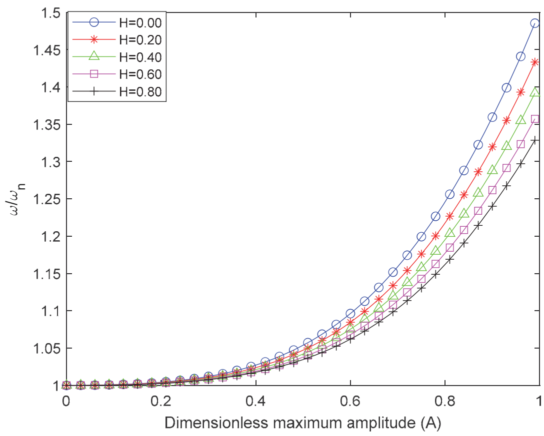


Figure 4: Effects of magnetic term on dimensionless amplitude-frequency ratio on simple-simple support

Figure 4, 5 and 6 shown impacts of magnetic term on dimensionless amplitude-frequency ratio curve of stability analysis of single-walled carbon nanotube structure in magneto-thermal electrostatic environment under the influence of Casimir force on two elastic foundations. Figure 4 depicted that as magnetic term increases from zero to maximum, frequency-ratio decreases toward linear system. This shows that for SWCNTs structure to gain stability, magnetic term must be kept at maximum meanwhile, Figure 5 and 6 depicted that as magnetic term increases from zero to maximum, frequency-ratio decreases until there converged at a specific amplitude and thereafter, the magnetic term start increases again from the convergence point to maximum as frequency-ratio decrease toward linear system. These show that for SWCNTs structure to gain stability, magnetic term must be kept at both minimum and maximum under the influence of Casimir effect for both clamped-clamped and clamped-simple supports.

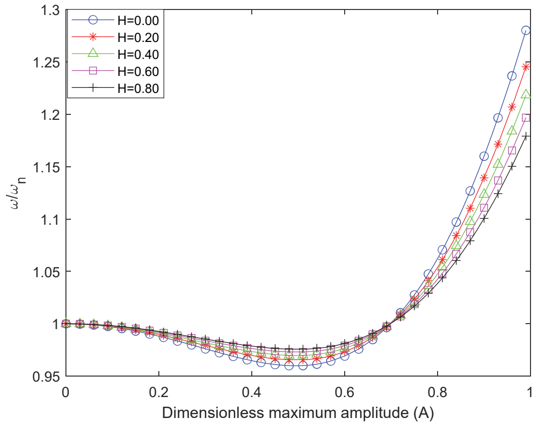


Figure 5: Effects of magnetic term on dimensionless amplitude-frequency ratio on clamped-clamped supports

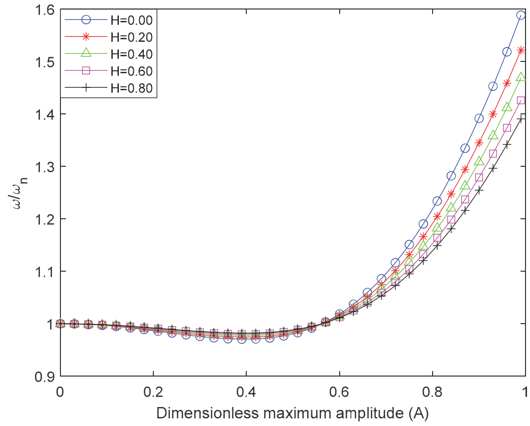


Figure 6: Effects of magnetic term on dimensionless amplitude-frequency ratio on clamped-simple support

Figure 7, 8 and 9 shown impacts of thermal term on dimensionless amplitude-frequency ratio curve of stability analysis of single-walled carbon nanotube structure in magneto-thermal electrostatic environment under the influence of Casimir force on two elastic foundations. Figure 7 depicted that as magnetic term increases from zero to maximum, frequency-ratio decreases toward linear system. This shows that for SWCNTs structure to gain stability, thermal term must be kept at maximum meanwhile, figure 8 and 9 depicted that as thermal term increases from zero to maximum, frequency-ratio decreases until there converged at a specific amplitude and thereafter, the thermal term start increase again from the convergence point to maximum as frequency-ratio decrease toward linear system. These show that for SWCNTs structure to gain stability, thermal term must be kept at both minimum and maximum under the influence of Casimir effect for both clamped-clamped and clamped-simple supports.

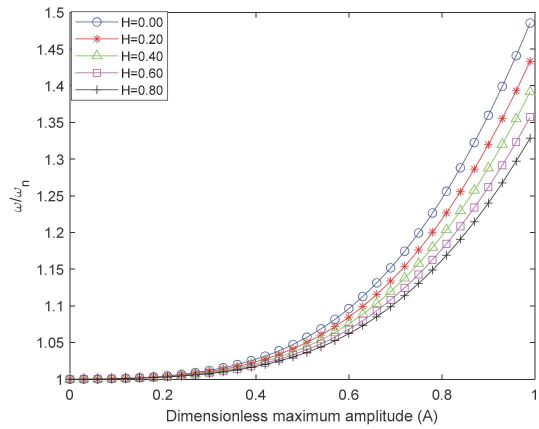


Figure 7: Effects of thermal term on dimensionless amplitude-frequency ratio on simple-simple support

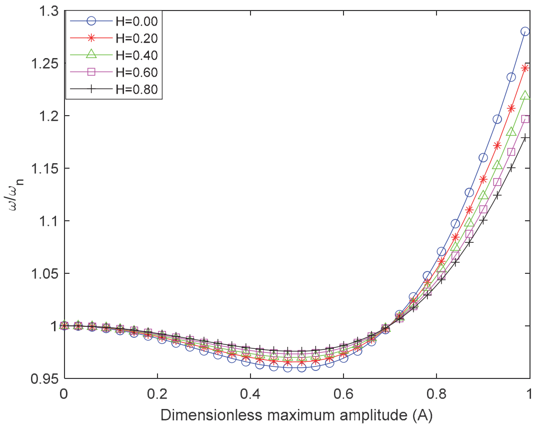


Figure 8: Effects of magnetic term on dimensionless amplitude-frequency ratio on clamped-clamped support

Figure 10, 11 and 12 shown impacts of linear Winkler-type elastic foundation on dimensionless amplitude-frequency ratio curve of stability analysis of single-walled carbon nanotube structure in magneto-thermal electrostatic environment under the influence of Casimir force on two elastic foundations. Figure 10 depicted that as linear Winkler-type elastic foundation increases from zero to maximum, frequency-ratio decreases toward linear system. This shows that for SWCNTs structure to gain stability, linear Winkler-type elastic foundation must be kept at maximum meanwhile, Figure 11 and 12 depicted that as linear Winkler-type elastic foundation increases from zero to maximum, frequency-ratio decreases minimally until there converged at a specific amplitude and thereafter, the linear Winkler-type elastic foundation start increase again from the convergence point to maximum as frequency-ratio decrease toward linear system. These show that for SWCNTs structure to gain stability, linear Winkler-type elastic foundation must be kept at both minimum and maximum under the influence of Casimir effect for both clamped-clamped and clamped-simple supports.

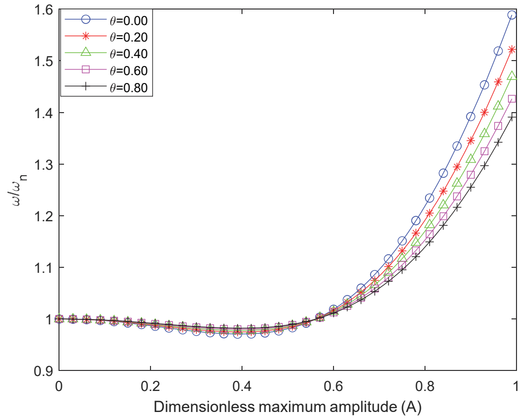


Figure 9: Effects of magnetic term on dimensionless amplitude-frequency ratio on clamped-simple support

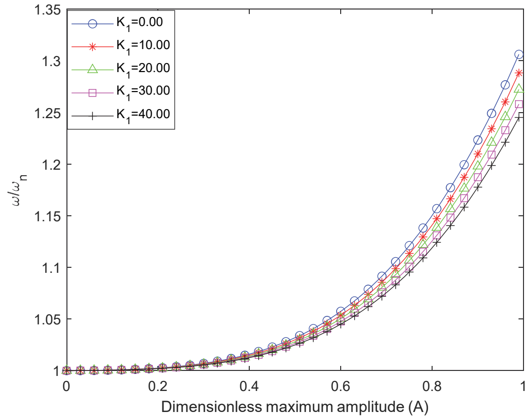


Figure 10: Effects of linear Winkler-type elastic foundation on dimensionless amplitude-frequency ratio on simple-simple support

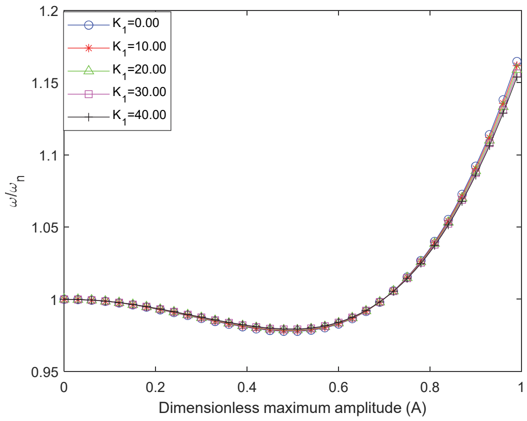


Figure 11: Effects of linear Winkler-type elastic foundation on dimensionless amplitude-frequency ratio on clamped-clamped support

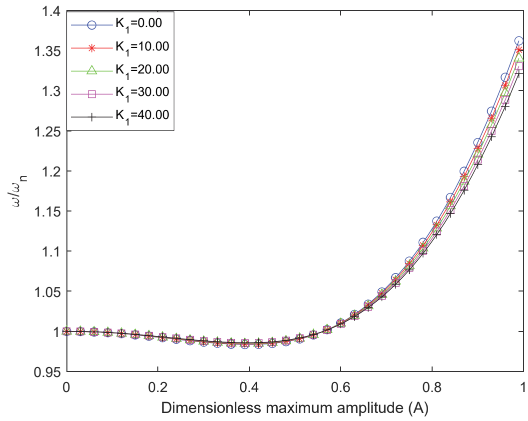


Figure 12: Effects of linear Winkler-type elastic foundation on dimensionless amplitude-frequency ratio on clamped-simple support

Figure 13, 14 and 15 shown impacts of Pasternak elastic foundation on dimensionless amplitude-frequency ratio curve of stability analysis of single-walled carbon nanotube structure in magneto-thermal electrostatic environment under the influence of Casimir force on two elastic foundations. Figure 13 depicted that as Pasternak elastic foundation increases from zero to maximum, frequency-ratio decreases toward linear system.

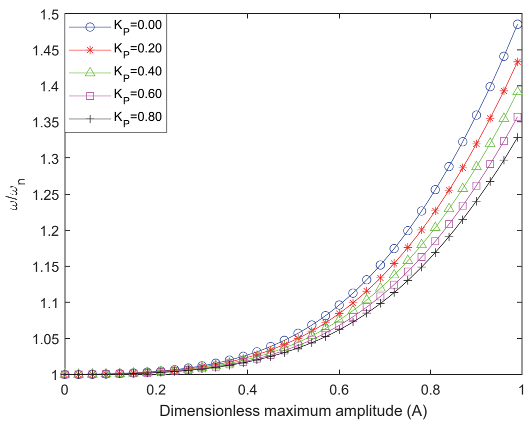


Figure 13: Effects of Pasternak elastic foundation on dimensionless amplitude-frequency ratio on simple-simple support

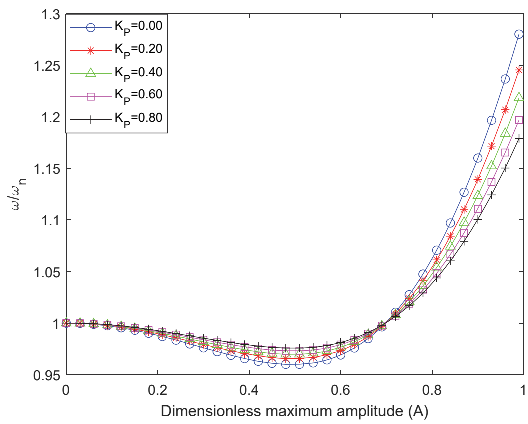


Figure 14: Effects of Pasternak elastic foundation on dimensionless amplitude-frequency ratio on clamped-clamped support

This shows that for SWCNTs structure to gain stability, Pasternak elastic foundation must be kept at maximum meanwhile, Fig.14 and 15 depicted that as Pasternak elastic foundation increases from zero to maximum, frequency-ratio decreases until there converged at a specific amplitude and thereafter, the Pasternak elastic foundation start increase again from the convergence point to maximum as frequency-ratio decrease toward linear system. These show that for SWCNTs structure to gain stability, Pasternak elastic foundation must be kept at both minimum and maximum under the influence of Casimir effect for both clamped-clamped and clamped-simple supports.

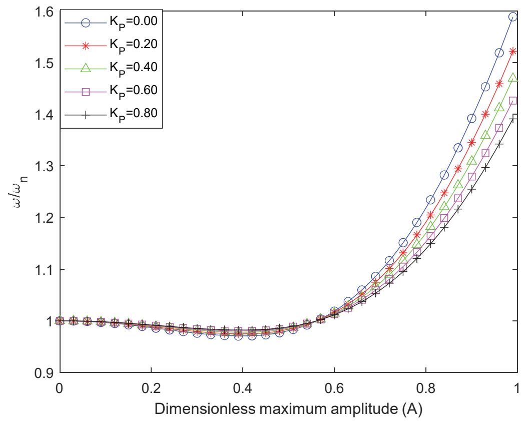


Figure 15: Effects of Pasternak elastic foundation on dimensionless amplitude-frequency ratio on clamped-simple support

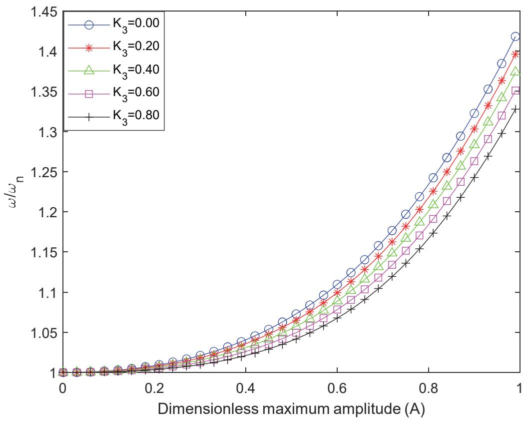


Figure 16: Effects of nonlinear Winkler-type elastic foundation on dimensionless amplitude-frequency ratio on simple-simple support

Figure 16, 17 and 18 depicted that as nonlinear Winkler-type elastic foundation increases from zero to maximum, frequency-ratio decreases toward linear system. This shows that for SWCNTs structure to gain stability, nonlinear Winkler-type elastic foundation must be kept at maximum.

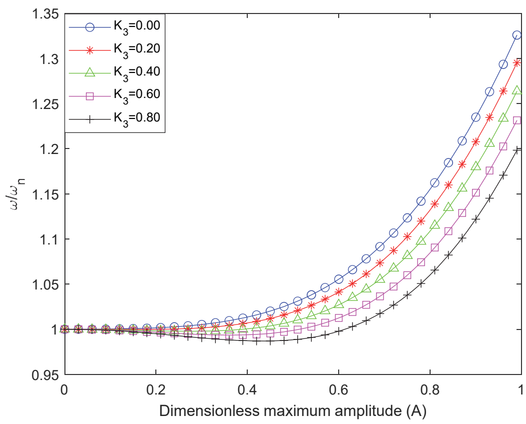


Figure 17: Effects of nonlinear Winkler-type elastic foundation on dimensionless amplitude-frequency ratio on clamped-clamped support

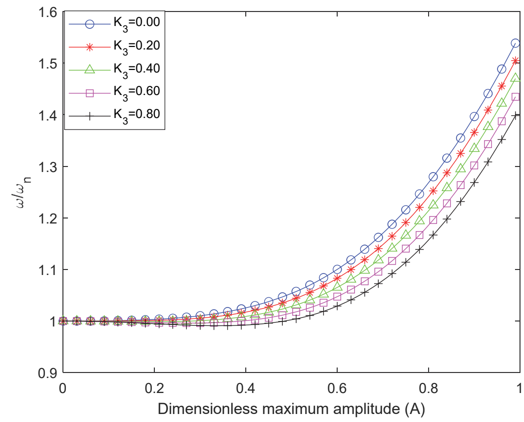


Figure 18: Effects of nonlinear Winkler-type elastic foundation on dimensionless amplitude-frequency ratio on clamped-simple support

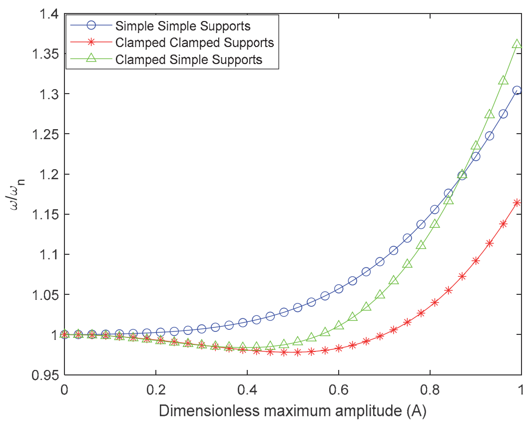


Figure 19: Effects of different boundary conditions on dimensionless amplitude-frequency ratio

Figure 19 shown impacts of different boundary conditions on dimensionless amplitude-frequency ratio curve of stability analysis of single-walled carbon nanotube structure in magneto-thermal electrostatic environment under the influence of Casimir force on two elastic foundations. Figure 19 depicted that clamped-simple supports has highest frequency ratio after experienced sudden decreases and merged with clamped-clamped until amplitude of 0.4nm before divergence. The clamped-clamped supports have a lowest frequency ratio. Therefore, this reveal that in selecting elastic foundation type, clamped-clamped support exhibit best foundation type with lowest frequency ratio and can be used to control stability of any foundation.

Figure 20 and 21 shown influence of Casimir force and Electrostatic force on time-deflection curve of single-walled carbon nanotube structure in magneto-thermal-electrostatic environment under the influence of Casimir force on two elastic foundations. The two-force concentrated at the compression zone as way of improving serviceability, long term deflections and to provide support for stirrups throughout the beam.

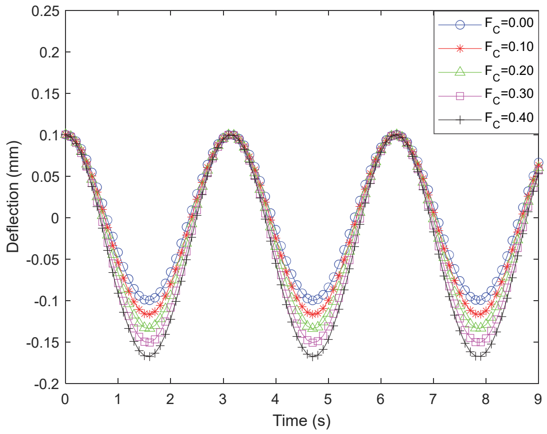


Figure 20: Effects of Casimir force on dimensionless Time-Deflection curve

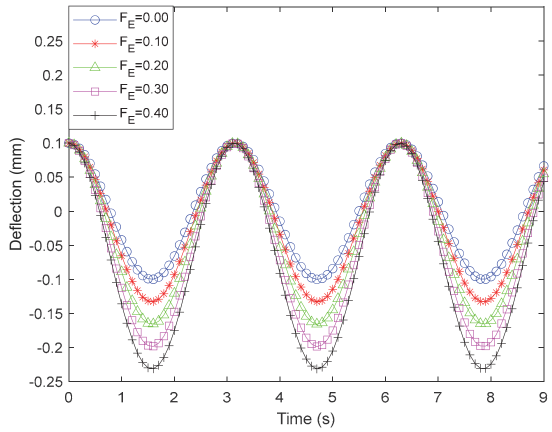


Figure 21: Effects of Electrostatic force on dimensionless Time-Deflection curve

4. CONCLUSION

The discovery of carbon nanotube structure has renewed a major chapter in the field of mechanics, physics, chemistry and materials science owing to their high-quality possession of; excellent tensile strength, high conductivity, high aspect ratio, thermally and high chemical stabilities etc. In this study, dynamic and instability analysis of single-walled carbon nanotubes with geometrical imperfection resting on elastic medium in a magneto-thermal-electrostatic environment with impact of Casimir force. The nonlinear mathematical model is derived with the aid of Eringen nonlocal theory and Hamilton principle. The resulting partial differential equation of motion is converted to duffing equation using Galerkin decomposition method. Subsequently, the duffing equation is solved using homotopic perturbation method (HPM). The results obtained from the simulation shows that, the results obtain depicted that, the effects of magnet term, thermal and Pasternak type foundation on dimensionless amplitude-frequency ratio for clamped-clamped and clamped-simply supports make the investigation novelty as it can be used as reference in future.

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