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The electromagnetic field of a current conductor in the presence of lossy half-space

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ABSTRACT

In this paper, the electromagnetic field problem of a line current conductor in the presence of homogenous and isotropic lossy half-space is analyzed. The main difficulty in this analysis is to take correctly into account the influence of ground finite conduct since the use of integral transformations in Maxwell's equations leads to Sommerfeld integral, hard to evaluate even numerically. Charge Simulation Method, based on the equivalence theorem of different electromagnetic systems, is proposed here for the numerical approach. Two independent, equivalent systems created of appropriately chosen, shaped, and arranged fictitious sources are suggested for the determination of EM field components, in the air and in the lossy half-space. Point-matching method based on the boundary conditions between media is applied for the determination of the fictitious source intensities. The proposed procedure has the advantage that it takes into account the influence of the finite ground conductivity, without calculation of Sommerfeld integral and it ensures excel-lent accuracy with a small number of fictitious sources. Also, the features of the proposed method are simple impel-mentation on a standard PC, fast and accurate simulation procedure, and easy generalization for some other cases of arbitrarily oriented wires of finite lengths above a lossy half space.

KEYWORDS

Lossy half-space, Line current conductor, Charge Simulation Method.

1. INTRODUCTION

Determination of the electromagnetic (EM) field of line current conductor located in the air, in the presence of homogeneous and isotropic lossy half-space, still attracts interest of many researchers [1-4], even though it has been more than a century since Sommerfeld published his solution of the Hertzian-dipole radiation above real ground [5]. His approach was to decompose the original field by an infinite number of plane waves and then apply the law of diffraction, reflection, and boundary conditions on a planar interface between the two media. This is far from a trivial task since it requires solving a semi-infinite integral with a very complex integrand, called Sommerfeld integral (SI), which describes the influence of a lossy half-space. Because SI is hard to evaluate both analytically and numerically, most of the methods for calculations have some limitations regarding under-integral parameters or calculation time.

One of the earliest attempts to solve the problem was based on transmission line analogies that led to useful results, particularly at low frequencies, see e.g. [6]. Although this method, known as Carson's method, does not provide a closed-form solution, it is still the standard method for determining the frequency-dependent impedance of overhead transmission lines. Over the past several decades, a number of studies for the calculation of magnetic field in the surrounding of an electric power transmission lines have been published [7–13], because of concerns about possible detrimental health effects from these kinds of fields [14-16]. Novel approach based on the use of artificial intelligence techniques was applied in [16-19] to estimate electric and magnetic fields around an overhead power transmission lines. Recently simple approximation that can be used for modeling of one type of Sommerfeld integrals typically occurring in the expressions that describe sources buried in the lossy ground, is proposed [20].

This problem has also been theoretically studied in the case of geophysical applications, where there is an assumption of a uniform ground and no propagation, which leads to a quasi-static approximation. Electromagnetic field at the surface of the ground in Fourier integral form is derived in [21]. The classical study in this area considered the EM fields of a phased line current over a conducting half-space [22]. A very interesting theoretical approach is exposed in [23], where the EM field components created at the ground surface by an overhead line current are expressed in terms of the Neumann and Struve functions, and derived in a new series expansions and computations.

There are many papers on the subject that deal with various facets of SI problem in analysis of dipole antennas in the presence of lossy half-space [24–34]. In some of them, different simpler models were proposed, and the results obtained by rigorous Sommerfeld's integral formulation were quoted [24-26], in the other the SI is approximately evaluated using different known functional solutions [27-29]. One interesting approach is proposed in [30] where the boundary element method, combined with the exponential approximation technique, is used for treating the integral equation for loaded wire above a lossy medium. In a number of papers [31, 32] a discrete complex image method, based on the replacement of the lossy half-space by a few complex images is used. Complex and exact modelling of the SI using images is given in [33]. In [34] the key to obtain the solution was to split the incident field into evanescent and traveling contributions. In most of these papers, the integral equations are approximately solved by the moment method, the so-called point-matching method [35,36].

The current paper describes the details of the numerical solution for EM field of current conductor above lossy half-space that relies on theoretical investigations of the Sommerfeld solution. An analytical solution developed in [37,38] for the problem of the current conductor above lossy half-space is based on transformations that substitute Somerfeld integral (obtained by the integral transformation method) with Hankel's function and their asymptotic expansion. In these references, the obtained analytical solutions are compared with the known results from the literature for the special case when ground is well conductive [39], and the obtained results show an excellent accordance expansion. This practically justifies the proposed presentation of the complex integrals by Hankel's functions and their asymptotic developments. Charge Simulation Method [40-42] based on the theorem of equivalence of different electromagnetic systems, is used for numerical calculations, due to the nature of the problem, system geometry and the influence of finite earth conductivity. Compared to other boundary methods, CSM is significantly faster and conceptually much simpler since numerical integration and singularisation of the integrand are not required.

Two independent, equivalent systems are suggested in this study. The first system is applied for EM field component determination in the air, and it consists of a primary line current conductor and several fictitious line current conductors. The second system is applied for field component determination in lossy half-space and it consists of fictitious line current conductors only. The positions of fictitious sources are not strictly defined in advance, which enables experienced researchers to choose the optimal configurations, which is the privilege of the method. EM field components contain Hankel's functions which represent linear combinations of Bessel's functions. Hankel's functions with different arguments, real in the air and complex in the ground, are obtained using recurrent relations [43]. The complex intensities of unknown currents are determined by the point-matching method, where boundary conditions in a finite number of points located on the boundary between two media have to be satisfied. Using this procedure, electromagnetic field components can be calculated, in any point above and below the ground surface. The problem solved here for certain cases is transformed into the application of various forms of image theorem, a special case of the equivalence theorem. The accuracy check of the applied method can be performed through some special regimes analysis [38]. The main contribution of the proposed procedure is that it takes into account the influence of the finite ground conductivity, without calculating Somerfeld integral and ensures excellent accuracy with a small number of fictitious sources. Finally, the advantage of the proposed procedure is that it can also be extended to the more demanding thin wire geometries which are commonly met in practice.

2. FORMULATION OF THE PROBLEM

2.1. Lonely Current Conductor

Thin, long conductor, with constant current amplitude, I, and frequency ω, located in homogeneous, isotropic, linear lossy space, with dielectric constant $\varepsilon = \varepsilon_0 \varepsilon_r$, magnetic permeability $\mu = \mu_0 \mu_r$ and specific conductivity σ is shown in Fig. 1. It is placed in cylindrical coordinate system (r, θ, z), along z-axes by direction $r = 0$, $\theta = 0$.

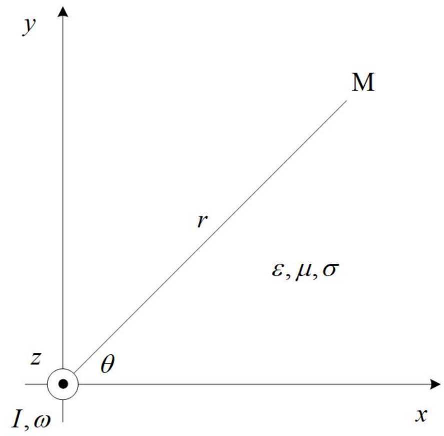

Figure 1: Lonely current conductor.

Electromagnetic field in surrounding of the conductor satisfies Maxwell's equations

$$\mathrm{rot}\mathbf{H} = \mathbf{J} + \underline{\sigma}\mathbf{E} \tag{1}$$

$$\mathrm{rot}\mathbf{E} = -\mathrm{j}\omega\mu\mathbf{H} \tag{2}$$

where

$$\underline{\sigma} = \sigma + \mathrm{j}\omega\varepsilon \tag{3}$$

is complex specific conductivity of space,

$$\mathbf{J} = J\hat{z}, J = I\delta(x)\delta(y) = \frac{I}{2r\pi}\delta(r) \tag{4}$$

where **J** is the current density, and δ denotes Dirac's functions.

The axial electric field component satisfies the wave equation

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}E_z}{\mathrm{d}r}\right) + k^2 E_z = \mathrm{j}\frac{\omega\mu I}{2\pi r}\delta(r) \tag{5}$$

where $k^2 = -j\omega\mu\underline{\sigma}$, and k is a complex propagation constant of lossy space.

This equation can be written in the form

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}E_z}{\mathrm{d}r}\right) + k^2 E_z = 0 \tag{6}$$

because only a lonely current conductor is considered here.

Solutions of this wave equation are Bessel's functions of the first kind, $J_0(kr)$, and of the second kind, $N_0(kr)$. Since only one solution that propagates from source to periphery is of importance, the linear combination of Bessel's functions, known as Hankel's function of the second kind is used:

$$H_0^{(2)}(kr) = J_0(kr) - jN_0(kr) \tag{7}$$

The solution of the wave equation, which at large distances satisfies radiation conditions,

$$\frac{\partial E}{\partial r} + E\left(\mathrm{j} + \frac{1}{2kr}\right) = 0,\ r \to \infty \tag{8}$$

has the following form

$$E_z = C H_0^{(2)}(kr) \tag{9}$$

where C is an unknown integration constant, obtained from the condition that low frequency magnetic field intensity

$$H_\theta = \mathrm{j}\frac{kC}{\omega\mu} H_1^{(2)}(kr) \tag{10}$$

tends towards a constant value

$$\lim_{\omega \to 0} H = \frac{I}{2r\pi} \tag{11}$$

Since it is known that

$$J_1(kr) \to \frac{kr}{2} \quad \text{and} \quad N_1(kr) \to -\frac{2}{\pi kr} \quad \text{for} \quad kr \to 0 \tag{12}$$

integration constant is obtained in the form

$$C = -\frac{\omega\mu I}{4} \tag{13}$$

Finally, electromagnetic field components are

$$E_z = -\frac{\omega\mu I}{4} H_0^{(2)}(kr) \tag{14}$$

$$H_\theta = -\mathrm{j}\frac{kI}{4} H_1^{(2)}(kr) \tag{15}$$

In the radiation zone, far field approximation of Hankel's functions can be expressed as

$$H_0^{(2)}(kr \to \infty) = \sqrt{\frac{2}{\pi kr}} e^{-\mathrm{j}\left(kr - \frac{\pi}{4}\right)} \tag{16}$$

In the immediate vicinity of the conductor, near field approximation for Hankel's functions can be taken,

$$H_0^{(2)}(kr \to 0) = 1 - \mathrm{j}\frac{2}{\pi}\ln(kr) \tag{17}$$

2.2. Line above lossy half-space

Thin, long conductor, with constant current amplitude, I, and frequency ω, located in air parallel above homogeneous, isotropic and linear lossy half-space, with dielectric constant $\varepsilon = \varepsilon_0 \varepsilon_r$, magnetic permeability $\mu = \mu_0 \mu_r$ and specific conductivity σ, is shown in Fig. 2. It is placed in a cylindrical coordinate system (r, θ, z), along z-axes, with $r=h$, $\theta=\pi/2$.

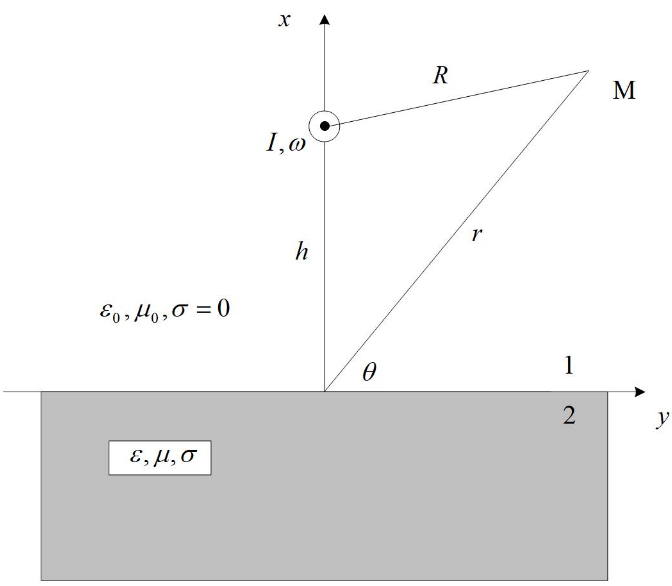

Figure 2: Linear conductor above lossy half –space.

The electric field has only axial component,

$$\mathbf{E} = E_z \hat{z}, \quad E_r = 0, \quad E_\theta = 0, \quad E_z = E_z(r,\theta) \tag{18}$$

while magnetic field has both radial and circumferential component, which can be expressed from Maxwell's equations

$$\mathbf{H} = H_r \hat{r} + H_\theta \hat{\theta}, \quad H_r = -\frac{1}{\mathrm{j}\omega\mu r}\frac{\partial E_z}{\partial \theta}, \; H_\theta = \frac{1}{\mathrm{j}\omega\mu}\frac{\partial E_z}{\partial \theta}, \; H_z = 0 \tag{19}$$

Axial component of electric field satisfies differential equation in the air

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial E_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 E_z}{\partial \theta^2} + k_0{}^2 E_z = \mathrm{j}\frac{\omega\mu_0 I}{2\pi r}\delta(r-h)\delta\left(\theta - \frac{\pi}{2}\right), \tag{20}$$

and wave equation in lossy half-space

$$\frac{1}{r}\frac{\partial E_z}{\partial r}\left(r\frac{\partial E_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 E_z}{\partial \theta^2} + k^2 E_z = 0 \tag{21}$$

where k_0 and k are the phase constant of air and lossy half-space, respectively,

$$k_0 = \omega\sqrt{\varepsilon_0 \mu_0} \tag{22}$$

$$k = \omega\sqrt{\varepsilon\mu}\sqrt{\frac{\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1}{2}} - \mathrm{j}\omega\sqrt{\varepsilon\mu}\sqrt{\frac{\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1}{2}} \tag{23}$$

and

$$R = \sqrt{r^2 + h^2 - 2rh\sin\theta} \tag{24}$$

2.3. . Method of solution

Charge Simulation Method (CSM) is proposed as a suitable numerical approach to this problem, because of its nature, system geometry and the influence of finite earth conductivity. As already stated in the Introduction part, CSM is based on the theorem of equivalence of different electromagnetic systems, [36,37,38].

Two independent, equivalent systems are suggested, with the geometries shown in Fig. 3. The first one is applied for field component determination in the air, and the second for field component determination in the lossy half-space. The first equivalent system consists of a primary, linear, very long current conductor, with uniform amplitude current *I*, and *N* fictitious, also linear, very long parallel conductors, with uniform amplitude currents I_{1n}, $n = 1,2,...,N$. The second equivalent system consists of *M* fictitious, linear, very long current conductors, with uniform amplitude currents I_{2n}, $n = 1,2,...,M$. The intensities of unknown currents are determined with the points adjustment method, satisfying boundary conditions in a finite number of points located on the boundary between two spaces.

The position of fictitious sources is not strictly defined, and it is up to the researcher to choose the optimal one. The variant, which is chosen as optimal, is shown in Fig. 3.

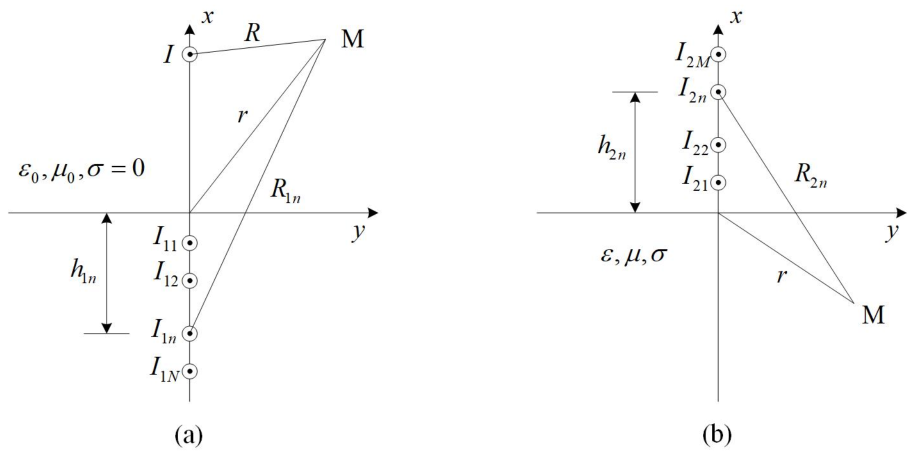

Figure 3: Equivalent systems for EM field calculation. (a) In the air. (b) In the lossy half-space..

Using the procedure given in section 2.1, EM components in lossy half-space are determined with

$$E_{2z} = -\frac{\omega\mu}{4}\sum_{n=1}^{M} I_{2n} H_0^{(2)}\left(kR_{2n}\right) \tag{25}$$

$$H_{2r} = -\mathrm{j}\frac{k}{4}\sum_{n=1}^{M}\frac{h_{2n}\cos\theta}{R_{2n}} I_{2n} H_1^{(2)}\left(kR_{2n}\right) \tag{26}$$

$$H_{2\theta} = -\mathrm{j}\frac{k}{4}\sum_{n=1}^{M}\frac{r-h_{2n}\sin\theta}{R_{2n}} I_{2n} H_1^{(2)}\left(kR_{2n}\right) \tag{27}$$

and, in the air

$$E_{1z} = -\frac{\omega\mu_0 I}{4} H_0^{(2)}\left(k_0R\right) - \frac{\omega\mu_0}{4}\sum_{n=1}^{N} I_{1n} H_0^{(2)}\left(k_0R_{1n}\right) \tag{28}$$

$$H_{1r} = -\mathrm{j}\frac{k_0 I}{4}\frac{h\cos\theta}{R} H_1^{(2)}\left(k_0R\right) - \mathrm{j}\frac{k_0}{4}\sum_{n=1}^{N}\frac{h_{1n}\cos\theta}{R_{1n}} I_{1n} H_1^{(2)}\left(k_0R_{1n}\right) \tag{29}$$

$$H_{1\theta} = -\mathrm{j}\frac{k_0 I}{4}\frac{r-h\sin\theta}{R} H_1^{(2)}\left(k_0R\right) - \mathrm{j}\frac{k_0}{4}\sum_{n=1}^{N}\frac{r-h_{1n}\sin\theta}{R_{1n}} I_{1n} H_1^{(2)}\left(k_0R_{1n}\right) \tag{30}$$

where

$$R_{1n} = \sqrt{r^2 + h_{1n}^2 - 2rh_{1n}\sin\theta} \quad \text{and} \quad R_{2n} = \sqrt{r^2 + h_{2n}^2 - 2rh_{2n}\sin\theta} \tag{31}$$

The intensities of unknown currents are determined using the point-matching method, satisfying boundary conditions in finite number of points located on the boundary between two spaces. Here, the boundary conditions for tangential components of the magnetic field and normal component of the magnetic flux densities are satisfied, so $H_{1r} = H_{2r}$ and $\mu_0 H_{1\theta} = \mu H_{2\theta}$. The total number of unknown currents is $L = N + M$. Boundary conditions are satisfied in L points on the boundary surface, in N points for the first boundary condition, and in M points for the second boundary condition.

3. NUMERICAL RESULTS

Using the procedure described in the previous section, electromagnetic field components can be calculated in any point that belongs to one, or another half-space, for different parameters of the lower one.

The assumption is that the conductor is infinite and thin, with a current amplitude 1 A, $\Psi = 0$, located on the distance of 1m from the lossy half-space. Investigations have shown that most types of lossy ground have $\varepsilon_r \geq 3$ and $\mu_r = 1$. For frequencies of a few MHz, the conductivity σ is in the range $0.001\,\text{S/m} \leq \sigma \leq 0.1\,\text{S/m}$. In this paper, three types of lossy ground are considered. The electrical characteristics of these types are shown in the Table 1.

Table 1: Electrical characteristics of three types of ground

Types of lossy ground	ε_r	σ [S/m]
dry earth (sand)	4	10^{-3}
wet earth	20	10^{-2}
fresh water	80	10^{-3}

Complex current intensities of the fictitious sources for each of these types of grounds are obtained. Also, the estimation of an optimal number of fictitious sources for each type of ground has been performed, based on the error analysis in satisfying the boundary conditions, with L ranging from 2 to 50.

For example, in the case of dry earth, with $f = 10\text{MHz}$, the error analysis shows that quite satisfactory accuracy is obtained even with $N = 5$, $M = 5$, that is to say only $L = 10$ fictitious sources being used. The obtained values for current intensities of fictitious sources for this case are presented in Table 2, in the air and the lossy ground. The rest of the results are also illustrated for this particular example.

Table 2: Calculated intensities of currents of fictitious sources in the air and in the lossy ground.

	In the air [A]		In the lossy ground [A]
I_{11}	$-0.02136+j0.03279$	I_{21}	$-0.02591+j0.05116$
I_{12}	$0.15192-j0.18325$	I_{22}	$0.22662-j0.47136$
I_{13}	$-0.50836+j0.40626$	I_{23}	$-0.96061+j2.02271$
I_{14}	$0.73974-j0.20655$	I_{24}	$1.87039-j3.83576$
I_{15}	$-0.35549-j0.16771$	I_{25}	$-0.35179+j2.56912$

Table 3 gives calculated tangential and normal components of the magnetic field in various points which belong to air, or to dry earth, again for the case of $L=10$ fictitious sources.

Table 3: Calculated tangential and normal components of the magnetic field in various points which belong to air, or to dry earth.

r[m]	θ[°]	H_r [A/m]	H_θ [A/m]
0.5	45	$0.20641+j0.00609$	$-0.05946-j0.007941$
0.8	80	$0.42965+j0.00101$	$-0.445626-j0.00824$
1.2	85	$0.28229+j0.00036$	$0.66106-j0.00714$
0.7	−75	$0.01507-j0.00029$	$0.09715-j0.00051$
0.4	−20	$0.10362+j0.00887$	$0.08276+j0.00111$
1.5	−60	$0.01468-j0.00214$	$0.06755-j0.01915$

The distribution of the intensity of the tangential component of the magnetic field in the air versus *r*, when θ takes values $20^\circ, 40^\circ$ and 60°, for the chosen case is presented in Fig. 4. The distribution of the intensity of the normal component of the magnetic field in the air versus *r*, when θ takes values $20^\circ, 40^\circ$ and 60° is presented in Fig. 5.

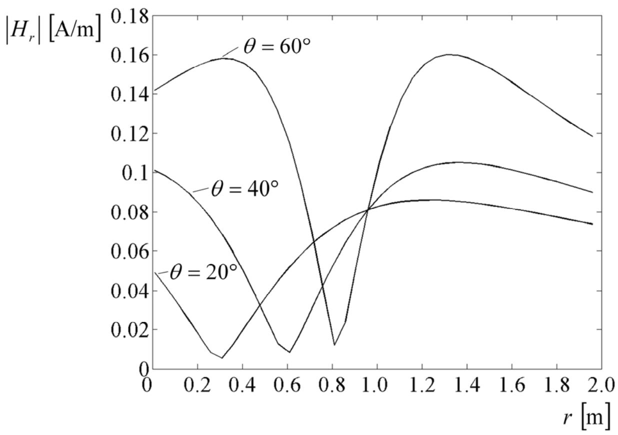

Figure 4: Tangential component of the magnetic field in the air versus *r*.

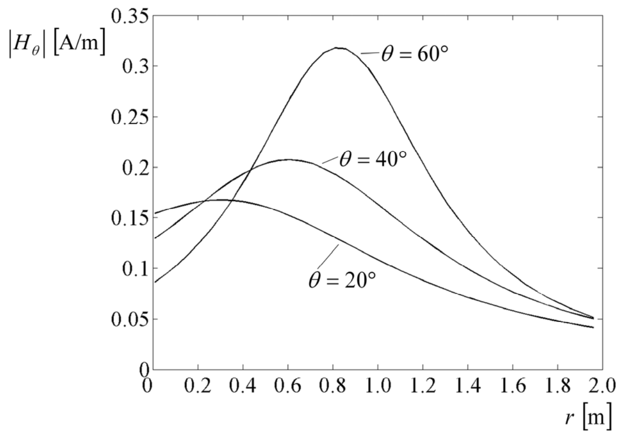

Figure 5: Normal component of the magnetic field in the air versus *r*.

The error analysis has been done for the remaining two ground types, and it has been estimated that the optimal number of fictitious sources for wet earth is $L=12$, and that it is necessary to increase the total number of fictitious sources for fresh water to at least $L=16$.

After the calculation of the electromagnetic field components, the resultant magnetic field is obtained as:

$$H_e=\sqrt{\mathbf{H}\mathbf{H}^*}=\sqrt{H_r^2+H_\theta^2} \tag{32}$$

The distribution of the intensity of the resultant magnetic field in the air versus *r*, for $\theta=45^\circ$, of the line source with sinusoidal current over the three types of imperfect ground is illustrated in Fig. 6.

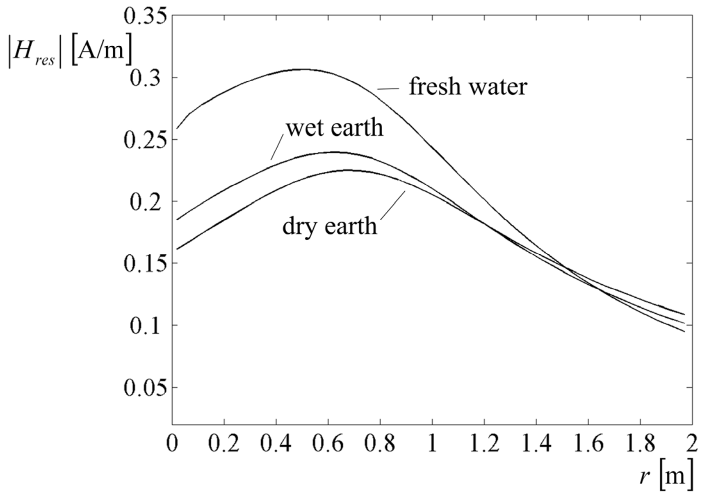

Figure 6: Resultant magnetic field versus *r*, and $\theta = 45^{\circ}$, for three types of imperfect ground.

4. CONCLUSION

The simulation model for analyzing a single current conductor placed above a homogenous and isotropic lossy half-space is proposed in this paper. The three cases when lossy half-space is substituted with the three different types of ground are particularly considered. CSM method, based on the theorem of equivalence of different electromagnetic systems and very applicable to systems that include two or more homogenous media, is proposed for numerical solution of the problem. This simulation model has a general character since it is not limited by the values of the parameters of the homogeneous ground. Also, the main contribution of this study is that proposed method provides taking into account the influence of the finite ground conductivity, without the calculation of Somerfeld integral, and ensures excellent accuracy with the small number of fictitious sources. Compared to other boundary methods, CSM is conceptually significantly faster and much simpler due to the fact that numerical integration and singularisation of the integrand are not required. Finally, the advantage of the proposed procedure is in the fact that this approach can also be generalized for the cases of various thin-wire geometries which are commonly met in practice.

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