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Contribution to the development of a method for optimal dimensioning of the steering arm in the conceptual design phase of a heavy motor vehicle

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ABSTRACT

The conceptual design represents the concretization of project requirements defined based on market data. This paper will focus on the steering system, specifically the steering arm. The parameters of the steering arm are not known in the initial design phase of a heavy motor vehicle, so an attempt is made here to define the necessary dimensions based on the minimization of its lateral vibrations, while simultaneously minimizing the mass and maximizing the critical buck-ling force. For this purpose, a simplified geometric, physical, and mathematical model of the steering arm, as well as the stochastic parameter optimization method and 2D Fourier transform, were used.

KEYWORDS

Heavy motor vehicle, Steering arm, Lateral vibrations, Mass, Critical buckling force, Two-parameter Fourier transformation.

1. INTRODUCTION

The heavy motor vehicle defined by the project task is further developed in subsequent design phases, where creative and intuitive approaches that played a significant role in the creation of the project task give way to logical and objective factors, calculations, measurements, shaping, evaluations of production and technological capabilities, etc. [1,2]. The conceptual design represents the first concretization of the project task [1].

This paper will focus on the steering system, specifically the steering arm.The assumption is that the project task defines the need to design a heavy motor vehicle for the market with a total mass of 11000 kg and a payload capacity of 4000 kg, with dimensions (length x width x height, mm): 6400 x 2500 x 3600, and a short cab. The engine is positioned at the front, and the vehicle is rear-wheel drive.The steering system of the newly designed vehicle must withstand rigorous exploita-tion conditions.

For illustration purposes, Figure 1a shows the diagram of the steering system, with the steering arm marked. The steering arm of the steering system will be dimensioned using optimization methods based on minimal lateral vibrations and mass, as well as maximum critical buckling force. It should be noted that optimization can be performed:

- theoretically, using mathematical models and dynamic simulations,
- experimentally, and
- in combination.

We note that experimental research is costly and often impractical in the initial design phase of a vehicle. Therefore, the approach of theoretical optimization has been accepted here, which required modeling of the steering arm, which will be further discussed.

2. METHOD

The aim of this paper is to explore the possibilities of applying optimization methods in selecting the dimensions of the steering arm in the conceptual design phase of a vehicle. The objective function should enable the minimization of lateral vibrations of the arm and its mass, while maximizing the critical buckling force.

2.1. Steering arm model

It was considered appropriate to first define in detail the load to which the steering arm is exposed during exploitation. For illustration, Figure 1b shows a possible position of the steering arm of heavy motor vehicles. It should be noted that option 1b-1 is used in heavy vehicles with rear-wheel drive, while 1b-2 is used in front-wheel drive vehicles.

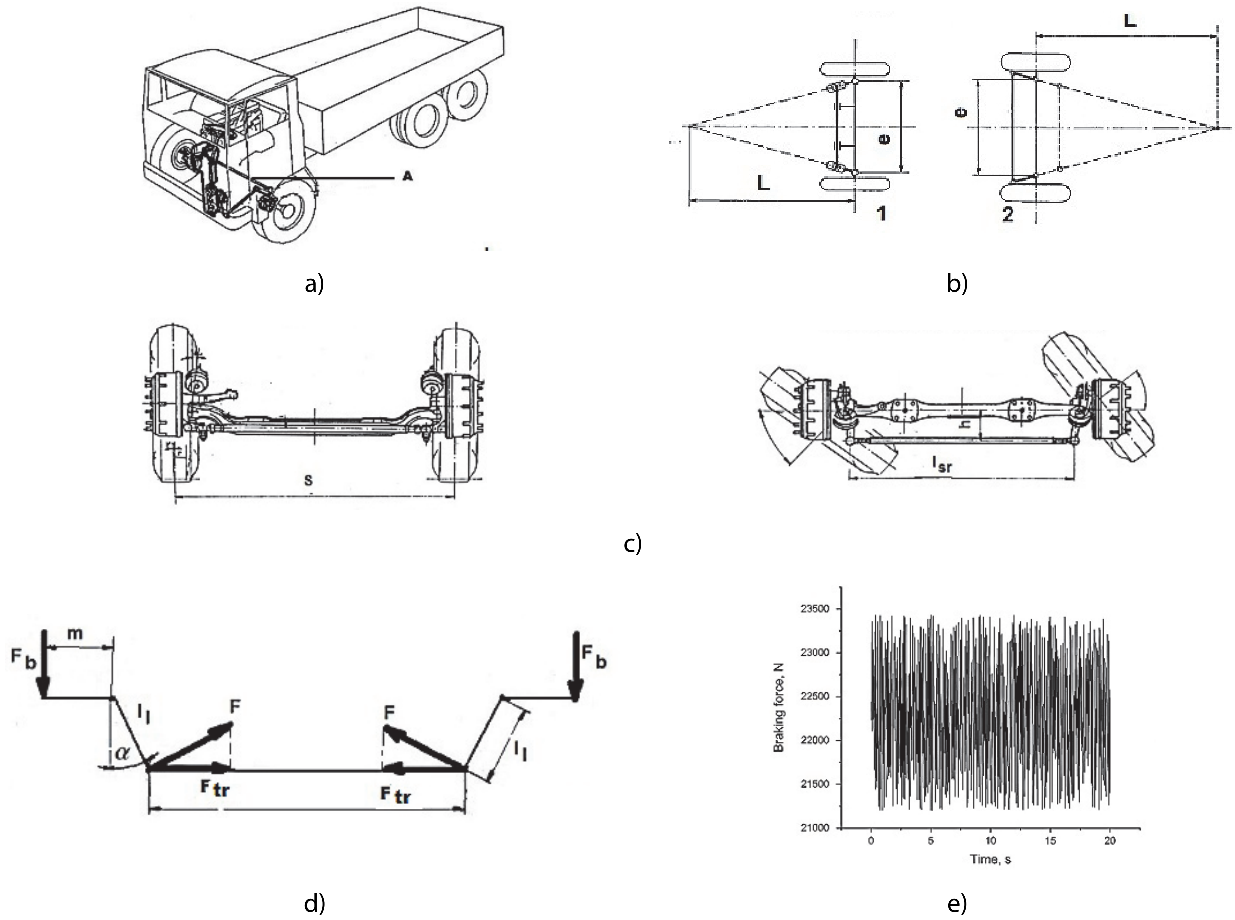

Figure 1: a) Position of the steering arm in the steering system of the subject heavy vehicle; b) possible positions of the steering arm; c) axle for FAP heavy vehicles; d) forces in the steering arm during vehicle braking; e) braking force

One of the methods for dimensioning the steering arm is based on the maximum force in the link during intense braking on a straight road [3]. In this case, the steering arm located behind the axle is subjected to compressive force and possible buckling, while if it is in front of the drive axle, it is subjected to tensile force. Since buckling is more critical from a dimensioning perspective, it will be discussed further.

It was considered appropriate to consider the front axle of the FAP vehicle for further analysis, designated as FAP 6.5, shown in Figure 1c. Figure 1d shows the forces acting on the steering arm during intense braking, so further discussion will focus on calculating the braking forces, or the force in the arm.

Based on the data on the selected axle, the maximum vertical force is Z=65000 N, α=12°, size l_l=228 mm, and the average distance from the center of the axle arm to the center of the wheel m=147 mm.

Taking into account [2], the total braking force during straight-line motion is given by the expression:

$$K = Z\varphi \tag{1}$$

where:

- Z - vertical force on the front axle, and
- φ - coefficient of adhesion - for braking on dry asphalt road, the adopted value is 0.7 [2].

To take into account the influence of micro-roughness on the coefficient of adhesion, in the absence of experimental data, it is assumed that it randomly varies with an amplitude of 2.5% from the adopted value. For illustration, Figure 1e shows the change in braking force for the assumed change in the coefficient of adhesion.

Considering Figure 1d, the magnitude of the force in the steering arm is given by the expression:

$$F_{tr} = \frac{K}{2}\frac{m}{l_l}\cos\alpha \tag{2}$$

It should be noted that the force F_{tr} represents the excitation function, Figure 2.

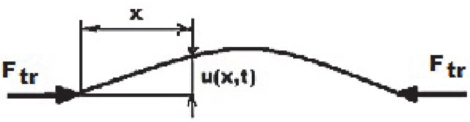

Figure 2: Model of the steering arm for analyzing its transverse vibrations

It was considered appropriate to model the steering arm as an elastic beam with a constant cross-sectional ring shape, loaded at the ends with variable axial forces, Figure 2. When defining a model to describe the lateral vibrations of the steering arm, the following assumptions were made [4,5]:

- the curvature of the arm is small during lateral vibrations,
- the influence of torsional and other dynamic loads is neglected,
- the cross-section of the arm is a pipe with constant radii,
- the mass of the arm is continuously distributed along its length, and
- the influence of friction and clearances in the arm joints is neglected.

Given that the partial differential equation describing the transverse vibrations of an elastic beam is described in detail in [4,5], it will not be repeated here, but only its final form will be given.

Based on the assumptions introduced, the forced transverse vibrations of the steering arm are described by the partial differential equation [4,5]:

$$\frac{\partial^2 u}{\partial t^2} - \frac{EI_x}{\rho A}\frac{\partial^4 u}{\partial x^4} = f(x,t) \tag{3}$$

where:

- u = u(x, t) - transverse vibrations of the steering arm,
- x - coordinate along the length of the arm,
- f(x, t) =F_{tr} - excitation function originating from intense vehicle braking,
- t - time,
- I_x - moment of inertia of the cross-section of the arm,
- ρ - material density,
- E – Young's modulus of elasticity, and
- A - cross-sectional area of the arm.

The area and moment of inertia of the annular cross-section of the steering arm are given by the expression:

$$A = \pi(R^2 - r^2); \quad I_x = \frac{\pi}{4}(R^4 - r^4) \tag{4}$$

where:

- R - outer, and
- r - inner radius of the arm tube.

As known [4,5], to find the general solution of the partial differential equation (3), it is necessary to know the boundary and initial conditions. In this specific case, both ends of the steering arm are connected by a spherical bearing, so the torques at the ends are equal to zero, and the forces due to vibrations are equal to the disturbance forces, i.e.:

$$-EI_x \frac{\partial^2 u(0,t)}{\partial t^2} = 0; \quad -EI_x \frac{\partial^2 u(l_s,t)}{\partial t^2} = 0$$
$$EA\frac{\partial u(0,t)}{\partial x} = F_{tr}; \quad -EA\frac{\partial u(l_s,t)}{\partial x} = F_{tr} \tag{5}$$

where:

- F_{tr} - force at the ends of the arm.

For dynamic simulation, it is assumed that the displacements at the spherical joints are zero at the initial moment, i.e.:

$$u(x,0) = 0; \quad u(l_s,0) = 0 \tag{6}$$

The disturbance force is encompassed by the boundary conditions (5). It should be noted that in solving the partial differential equation (3), it is sometimes necessary to introduce additional initial conditions [6].

Partial differential equations (3), in closed form, with boundary and initial conditions (5), (6) and a disturbing force (2), can only be sought in the case of harmonic excitation [6], so an attempt was made to solve it using the Wolfram Mathematica 13.2 sofware [6]. However, difficulties arose with scrolling numerical data and the fact that the mentioned software only solves partial differential equations up to the second order, so it was decided to solve the problem numerically [7], using the finite difference method. Since this procedure is known from [7], there will be no further discussion about it here, and the differential equation was solved using the developed Pascal software.

2.2. Dimensioning of the steering arm

In the following text, there will be more words about the optimal dimensioning of the steering arm. It should be noted that various methods are used in practice for this purpose, and here the "stochastic parametric optimization" method will be applied. As is known, the "stochastic para-metric optimization" method is used in optimizing the oscillatory parameters of motor vehicles and is based on nonlinear programming methods [8,9]. Since there are constraints on the design parameters in the optimization process, the problem is solved by introducing "external" or "internal" penalty functions [8].

In this specific case, the method of "stochastic parametric optimization" based on the Hooke-Jeeves method and "external" penalty functions was used for the selection of dimensions [8,9]. Since this optimization method is described in detail [9], it will not be discussed here. For illustration purposes, Figure 3 shows its block diagram, and the software is implemented in Pascal.

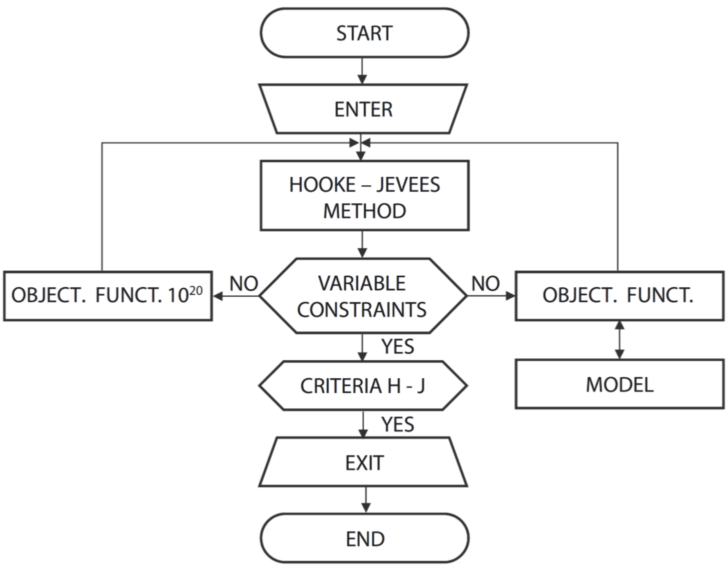

Figure 3: Block diagram of the optimization method

It was deemed appropriate to make the optimal choice of dimension parameters for the steering arm based on the conditions of minimal arm vibrations as an elastic system, its minimum mass, and maximum critical buckling force. With this in mind, the objective function of the form was used:

$$\Phi = \alpha_1 u_{RMS} + \alpha_2 m - \alpha_3 F_{kr} \tag{7}$$

where:

- α_1, α_2, α_3 - weight factors that define the influence rank of the variables on the objective function and allow for the conversion of variables that define sub-objectives into the same units. In the absence of recommendations for the selection of these coefficients, the case where they are equal to one was observed,
- u_{RMS} - values of the steering arm vibrations obtained by solving the partial differen-tial equation (3), and
- F_{kr} - critical buckling force of the arm.

The RMS transverse vibrations of the arm are calculated using the expression:

$$u_{RMS}{}^2 = \frac{1}{n_x n_t} \sum_{i=1}^{nx} \sum_{j=1}^{nt} u(i,j)^2 \tag{8}$$

where:

- u(i,j) - transverse vibrations of the arm,
- nx - number of points along the x-axis, and
- nt - number of points along the t-axis.

The mass of the steering arm is calculated based on the dimensions of the cross-section and the its length, using the expression:

$$m = \rho A l_{tr} \tag{9}$$

where:

- A - cross-sectional area of the beam (expression 4),
- ρ - material density, and
- l_{tr} - length of the arm.

The critical buckling force of the steering arm was calculated using the following expression [10]:

$$F = \pi^2 \frac{E I_{min}}{L_o^2} \tag{10}$$

where:

- L_o - buckling length (in our case, equal to the length of the arm ltr for both hinged ends of the arm), and
- I_{min} - minimum moment of inertia of the cross-section of the arm, given by expression (4).

During the process of optimal selection of the geometric parameters of the arm, based on the data of the adopted axis, its length l_{tr}=1560 mm, and the boundary values of the pipe radius of the arm were used: $20 \le R \le 25$; $10 \le r \le 20$. By introducing the optimizing parameter x[i], i=1,2, instead of R and r, with corresponding boundary values $x_u[i]$, $x_l[i]$, i=1,2, the objective function depends on two optimizing parameters and it has multiple local minima and only one global minimum [8,9]. Considering that, in practice, finding the global minimum is achieved by starting the optimization process with multiple initial values of the optimizing parameters [9], it was deemed appropriate to start with three initial values of these parameters in this case:

$$x = 0.5(x_u[1] + x_u[2])$$

$$x = 0.8 x_u[1]; x = 0.8 x_u[2]$$

$$x = 1.2 x_l[1]; x = 1.2 x_l[2]$$

The optimization was performed on a Pentium 4 computer (Intel 2.4 GHz, 9 GB RAM), and the iterative process was automatically terminated when the difference between two adjacent values of the objective function was 10-15. The optimization time for each combination was about 15 minutes, and the calculated parameters are shown in Table 1.

Table 1: The calculated parameters

Initial values	**R** [mm]	**r** [mm]	**Objective function** Φ	**Number of iterations**
$0.5(x_u[1]+x_u[2])$	22.50	10.00	$-1.64726 \cdot 10^5$	646
$0.8x_u[1];0.8x_u[2]$	20.00	16	10^{20}	313
$1.2x_l[1];1.2x_l[2]$	24.00	10.00	$-2.15216 \cdot 10^5$	580

3. DATA ANALYSIS

By analyzing the data from Table 1, it can be concluded that the lowest value of the objective function was obtained when the initial values of the optimizing parameters were in the middle of their boundary values. Therefore, for practical reasons, this combination is adopted as the global minimum. It should be noted that in the case of starting the optimization process with $0.8x_u$, the values of the objective function obtained were 10^{20}, which indicates that the optimizing parameters were outside the adopted boundary values.

By using the software developed in Pascal, the maximum force in the joint F_{trrmax}= 15063, N was calculated, and based on equation (10), the critical buckling force, assuming both ends are free, is F_{cr} = 164742, N. The safety factor is given by the expression [10]:

$$\nu = \frac{F_{cr}}{F_{tr\,\max}} = \frac{164742}{15063} = 10.93 \tag{11}$$

The value of the safety factor is significantly greater than 3.5 [10], so it can be argued that the optimized parameters are acceptable for this design phase. It should be noted that in the later stages of vehicle design, the safety factor can be reduced by changing the thickness of the pipe wall.

It is considered useful to calculate the slenderness of the steering arm for optimal parameters - (previously, the surface area and minimum moment of inertia were calculated: A=1276 mm^2; I_{xmin}=1.934·10^5 mm^4) [10]:

$$\lambda = \frac{l_{tr}}{\sqrt{\frac{I_x}{A}}} = \frac{1560}{\sqrt{\frac{1.934\cdot 10^5}{1276}}} = 126.71 \tag{12}$$

Since the slenderness of the steering arm is greater than 100 (for steel), it can be argued that it is acceptable to use Euler's theory [10]. During the optimization process, a dynamic simulation was performed for the steel arm using the following data: E=2.1·10^5, N/mm^2; ρ=8·10^{-6} , kg/mm^3; n_x=1000; h_x=1.56, mm; n_t=1000; h_t=0.02 s. The chosen values for the number of points and discretization steps allowed for reliable results for the parameter x: 0.00064 to 0.32,1/mm and t: 0.04 to 25 Hz [11]. It was deemed appropriate to also perform an analysis of the transverse vibrations of the steering arm for the optimal parameters, which are shown in Figure 4, for illustration.

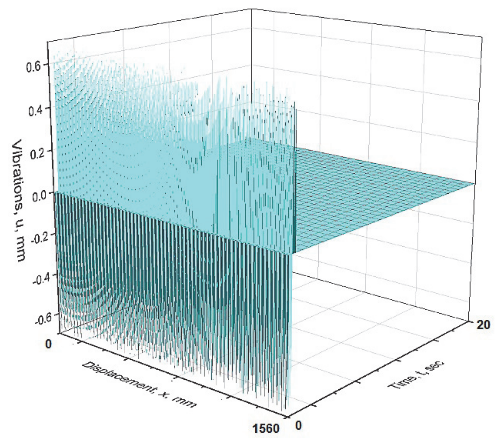

Figure 4: Transverse vibrations of the steering arm

From Figure 4, it can be seen that the transverse vibrations of the steering arm depend on the position along the arm and the time of observation of the wave motion. Due to the random nature of the excitation forces, the transverse vibrations have a random character, which is in accordance with [4,5].

For frequency analysis, it is necessary to apply the 2D Fourier transformation (the author developed software in Pascal [12]). However, considering the available commercial software on the market, it was considered appropriate to use Origin 8.5 [13] for further analysis, as potential users will have easier access to this software. Using the mentioned software, the sizes and phase angles of the 2D Fourier transform were calculated, and the results are shown in Figures 5 and 6, for illustration.

By analyzing the data from Images 5 and 6, it can be determined that the vibrations (spectrum magnitudes and phase angles) vary along the length of the steering arm and depend on time. This fact confirms the theoretical knowledge about transverse vibrations of an elastic beam [4].

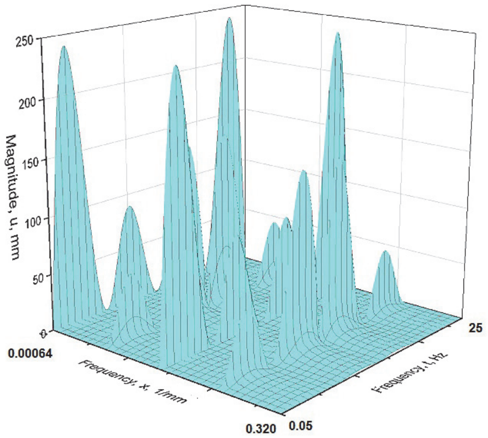

Figure 5: The spectrum magnitude of transverse vibrations of the steering arm

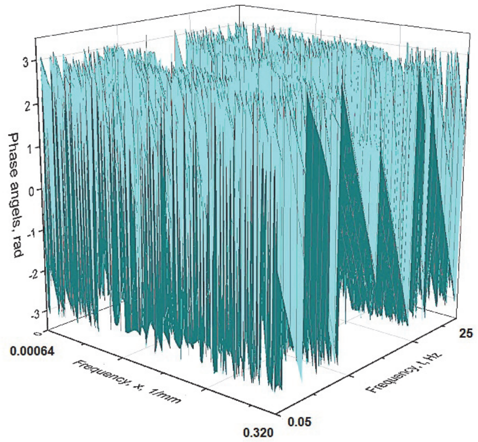

Figure 6: The phase angles of the spectrum of transverse vibrations of the steering arm

It should be noted that the calculated dimensions are preliminary and can be changed in a later stage, especially when adopting a standard tube for the steering arm. This can be done at the expense of reducing the safety factor, with a final calculation verification using the finite element method and experimentation, which will not be discussed here [14]. It should be noted that based on the adopted standard dimensions, and analysis of the resonance of the steering arm must be performed and compared with the frequencies of the harmonics obtained through 2D Fourier transform. The goal is to avoid overlap between the arm resonance and wave frequencies, which will not be discussed further here.

It is also worth noting that there are no explicit procedures for calculating errors in spectral analysis for 2D Fourier transform, as in the case of 1D Fourier transform [11]. Keeping this in mind, as well as the fact that the goal of this work is to illustrate the potential application of 2D Fourier transformation in the analysis of transverse vibrations of the steering arm, statistical errors were not calculated.

4. CONCLUSION

The developed procedure, based on the analysis of transverse vibrations, mass, and critical buckling force of the steering arm, allows for defining its dimensions in the initial stage of vehicle design. In further development of the project, based on the defined parameters of the steering arm, more detailed calculations can be performed, possibly using the finite element method. The conducted analyses have shown that the use of 2D Fourier transform is desirable for analyzing the transverse vibrations of the steering arm, which specifically refers to the analysis of its resonant frequencies.

REFERENCES

[1] M. Demić, “Osnovi projektovanja teretnih motornih vozila”, Mašinski fakultet, Kragujevac (Serbia), (1994)

[2] D. Simić, “Dinamika motornih vozila”, Naučna knjiga, Belgrade (Serbia), (1988)

[3] M. Minić, “Sistemi za upravljanje teretnih vozila : teorija, proračun, konstrukcija, ispitivanje”, ABC Glasnik, Beograd (Serbia), (1992)

[4] S. S. Rao, “Vibration of continuous system”, John Wiley and Sons, New Jersey (USA), (2007)

[5] S. L. Sobolev, “Partial differential equations of mathematical physics”, Elsevier (UK), (2016)

[6] https://Reference.Wolfram.com/language/tutorial/NDSolverPDE.html

[7] R. G.Stanton, ”Numerical methods for science and engineering”, Prentice-Hall, New Jersey (USA), (1961)

[8] B. D. Bunday, “Basic optimization methods”, Edvard Arnold, London (UK), (1984)

[9] M. Demić, “Optimization of vehicles elasto-damping elements characteristics from the as-pect of ride comfort”, Vehicle System Dynamics, Vol. 23(1), pp. 351-377, https://doi.org/10.1080/00423119408969066, (1994)

[10] S. Timoshenko, “Strength of materials, Part 2: Advanced theory and problems”, D. Van Nostrand Company Inc, New York (USA), (1941)

[11] J. S. Bendat and A. G. Piersol, “Random data-analysis and measurement procedures”, John Wiley and Sons, New Jersey (USA), (2000)

[12] M. Demić, "Application of multi-parameters frequency analysis in experimental identification of vibration parameters in motor vehicles", Mobility & Vehicle Mechanics, Vol. 49(1), pp. 25-37, https://doi.org/10.24874/mvm.2023.49.01.03, (2023)

[13] Software: OriginPro 8.5.0 SR1 b161 (1991-2010), Origin Lab Corporation, Northampton, MA. 01060 USA.

[14] K.J. Bathe, "Finite Element Procedures in Engineering Analysis", Prentice Hall, New York (USA), (1982)