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Contribution to the research of the possibility of application of two-parameter frequency analysis in the experimental identification of torsional vibration parameters of elastic shafts in vehicles power transmissions

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ABSTRACT

During exploitation, motor vehicles are exposed to vibrational loads that lead to fatigue of users and materials of their aggregates. Therefore, vibrations must be studied even in the earliest stages of design, using mathematical models, experiments, or their combinations. In theoretical considerations, vibrations of concentrated masses are usually observed, although, with the recent development of numerical methods (especially finite element methods), attention is also being paid to vibrations of vehicle elastic systems. This usually involves idealizations, especially regarding exploitation condtions and interconnections of motor vehicle aggregates.

In this paper, an attempt has been made to develop a method for identifying real vibrational loads of elastic power transmission shafts in vehicles (engine, rail, etc.) under exploitation conditions. Namely, 2D Fourier transformation was used for two-parameter frequency analysis. The possibilities of applying the procedure were illustrated on an idealized elastic torsionally loaded shaft. The conducted research showed that two-parameter frequency analysis can be used in generating torsional vibrations of elastic shafts of vehicles in laboratory conditions.

KEYWORDS

Vehicle, Elastic shaft, Torsional vibrations, Two-parameter frequency analysis

1. INTRODUCTION

During exploitation, motor vehicles are exposed to vibrational loads that lead to fatigue of users and materials of their aggregates. Therefore, vibrations must be studied even in the earliest stages of design, using mathematical models, experiments, or their combinations. In theoretical considerations, vibrations of concentrated masses are usually observed, although, with the recent development of numerical methods (especially finite element methods), attention is also being paid to vibrations of elastic vehicle systems. This usually involves idealizations, especially regarding exploitation conditions and interconnections of motor vehicle aggregates [1].

The specificity of the exploitation conditions of vehicles is their random nature [1], which significantly complicates theoretical considerations using models which makes experiments practically indispensable. Despite significant progress in the development of software for the automatic design and calculation of vehicles [4], the final judgment on their characteristics is based on experimental research. Hence, experimental methods are still significant today. Regarding elastic vehicle power transmission shafts that are subjected to torsional vibrations, there is often a problem in identifying the parameters of these vibrations.

In this regard, methods for their identification have been developed, such as modal analysis [5-10]. In laboratory conditions, vibration modes are practically determined. However, a problem arises in the case when real exploitation conditions are necessary to generate torsional loads on test equipment since modal analysis does not provide sufficient possibilities for generating these signals. Therefore, it was deemed appropriate to develop a procedure for identifying the parameters of torsional vibrations of elastic vehicles power transmission shafts, which would enable their generation under laboratory conditions.

One possibility is frequency analysis using Fourier transformation, which enables the determination of the frequency content of the signal by calculating spectra magnitudes and phase angles [11]. Data on the spectra magnitudes and phase angles, using inverse Fourier transform, enable the generation of the original, time-dependent signal, which is per-formed routinely in cases where the signal depends only on time [11].

However, vibrations of elastic systems depend on several parameters (dimensions and time), leading to the conclusion that multi-parameter Fourier transformation must be used [12-16]. In the case of idealized (neglecting other types of vibrations) torsional vibrations of elastic shafts, the so-called two-parameter Fourier transformation (2D) can be used [13,14], as the vibration change along the length of the shaft and time.

This paper analyzes the possibilities of using the two-parameter Fourier transform to create conditions for studying vibrations of elastic power transmission shafts in vehicles under laboratory conditions. Therefore, a general expression for the Fourier transform will be given in the case of multiple variables [17]:

$$F(\xi_1,\xi_2....\xi_n)=\int_{R^n} e-2\pi i(x_1\zeta_1+x_2\zeta_2+...+x_n\zeta_n)* \\ f(x_1,x_2....x_n)dx_1dx_2****dx_n \tag{1}$$

where:

- $f(x_1,x_2....x_n)$ - a function of n variables,
- $x_1,x_2....x_n$ - variables,
- $\xi_1,\xi_2....\xi_n$ - circular frequency,
- $\int_{R^n}$ - multiple integral (for 2D - double, 3D - triple, etc.).

2. METHOD

As previously noted, this paper aims to explore the possibility of using two-parameter frequency analysis (2D Fourier transform) for identifying parameters of torsional vibrations of elastic vehicles power transmission shafts. Considering that elastic elements and assemblies of vehicles can be simplified by modeling them as elastic-continual systems, it was deemed appropriate to explain the process using torsional vibrations of power transmission shafts of vehicles and 2D Fourier transform.

In the absence of experimental data on registered torsional vibrations of the shafts, the method was illustrated using data obtained from a mathematical model, i.e., dynamic simulation. As known, vibrations of elastic elements describe partial differential equations [13,14]. Since the partial differential equations that are described with torsional vibrations of elastic shafts are described in detail in [13,14], it will not be done here. Instead, its final form will be presented for further consideration, as shown in Figure 1.

In defining the model of torsional vibrations of an elastic shaft, the following assumptions were made:

- forces Fo1 and Fo2 are constant, and the influence of non-uniformity of the rotational movement of the shaft is covered by a special function,
- disturbance torque, for the sake of simplification of the problem, the influence of radial forces that cause transverse vibrations of the shaft will be neglected - this was possible to do because torsional vibrations originating from real loads are registered in operational conditions [16], and in this paper, only the idea of the method is considered.

- during the rotation of gears in oil, resistance torque appears due to viscosity, which is proportional to the speed of torsional vibrations - the influence of oil on torsional vibrations of the shaft is neglected, gears are rigid and have the same vibrations as the ends of the shaft, and they are replaced with discs,
- gears with straight teeth are observed, so axial loads that are transferred to bearings are omitted, and
- the shaft is elastic and has a constant circular cross-section.

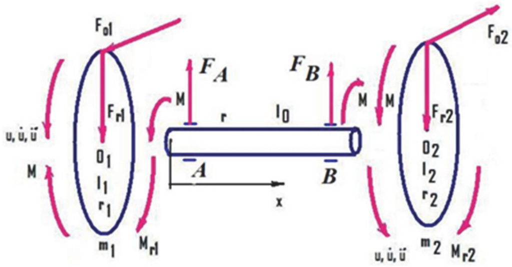

Figure 1: Force and torque diagram of elastic shaft

Taking into account the introduced assumptions, forced torsional vibrations of the elastic shaft [13,14] are described by a partial differential equation:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} + f(x,t) \qquad (2)$$

where:

- $u(x,t)$ - torsional vibrations of the shaft,
- x - coordinate along the length of the shaft,
- $f(x,t)$ - disturbance torque originating from non-uniform coupling of gears at the ends of the shaft,
- t - time.

$$c^2 = \frac{G}{P}$$

where:

- G - shear modulus, and
- ρ - shift material density.

As it is known [13,17], to find the general integral of the partial differential equation (2), it is necessary to know the boundary and initial conditions.

The boundary condition is derived from the condition of equality of the torsional torque of the left end of the shaft and the sum of torques that describe the dynamic behavior of the left gear. Using the adopted assumptions is obtained:

$$I_2 u = -F_{o2} r_2 - M_{r2} + M$$

where:

- $M = GI_0 \frac{\partial u(x,t)}{\partial x}$ - torque in the elastic shaft.

So it can be written as:

$$\frac{\partial u(x,t)}{\partial x} = (F_{o1} r_1 - k \frac{\partial u(x,t)}{\partial t} - I_1 \frac{\partial^2 u(x,t)}{\partial t^2}) \frac{1}{GI_0} \qquad (3)$$

Similarly, this can be done at the right end of the shaft:

$$I_1 u = F_{o1} r_1 - M_{r1} - M$$

$$\frac{\partial u(x,t)}{\partial x} = (F_{o2} r_2 + k \frac{\partial u(x,t)}{\partial t} + I_2 \frac{\partial^2 u(x,t)}{\partial t^2}) \frac{1}{GI_0} \qquad (4)$$

To define the boundary conditions of the left end of the shaft, x = 0 should be placed, and for the right end, x = L, where L is the length of the shaft.

The following abbreviations were used in expressions (3,4):

- F_{o1}, F_{o2} - circumferential forces on the gears,
- r_1, r_2 - radius of the gears,
- I_1, I_2 - axial moments of inertia of the gears defined by the expression:

$$I_j = \frac{m_j r_j^2}{2}, j = 1,2$$

where:

- mj - masses of the respective gears,
- rj -radius of the respective gears,
- Io - the polar moment of inertia of the circular cross-section of the elastic shaft given by the expression:

$$I_0 = \frac{\pi r^4}{2}$$

where:

- r is the radius of the shaft, and
- k is the coefficient that takes into account the friction due to the viscosity of the oil.

It was deemed appropriate to use the following forms of disturbance torque in the partial differential equation (2):

$$f(x,t) = a_m \sin(t)$$
$$f(x,t) = a_m \sin(x)$$
$$f(x,t) = a_m \sin(xt)$$
$$f(x,t) = a_m (rnd - 0.5)$$

where:

• a_m is the amplitude, and

• rnd - random numbers uniformly distributed in the interval [0,1].

The following initial conditions were assumed for analysis:

$$u(x,t) = 0$$
$$\frac{\partial u(x,t)}{\partial t} = 0 \qquad (5)$$

for t = 0.

The integral of the partial differential equation (2), with the boundary (3,4) and initial conditions (5) in final form, can only be searched in the case of harmonic excitation [13,14] .

Therefore, an attempt was made to solve it using the Wolfram Mathematica 13.2 software [17]. However, difficulties arose with listing numerical data, so it was decided to solve the problem numerically [18], using the finite difference method. Since this process is known from [18], it will not be discussed here, and the problem was solved using the software developed in Pascal.

Dynamic simulation was performed for a steel elastic shaft, using the following data: G = 8 * 10^4, N/mm^2; ρ = 8 * 10-6, kg/mm^3; m_1 = 2, kg; m_2 = 1.5, kg; r_1 = 100, mm, r_2 =120, mm; r = 15, mm; k = 1, Nm s/rad; F_{o1} = 2000, N; F_{o2} = 1850, N; n_x = 256, h_x = 2, mm; n_t = 256; h_t = 0.01, s; a_m = 20, Nm.

Since torsional vibrations of the elastic shaft depend on two parameters, it is necessary to use 3D graphics for their graphical representation.

For illustration, the results of numerical integration of the partial differential equation (2), with boundary conditions (3,4) and initial conditions (5), are shown in figures 2-5.

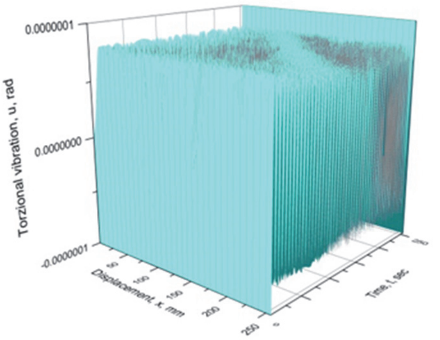

Figure 2: Torsional vibrations of the shaft for the excitation function u(x,t)= a_m*sin(t).

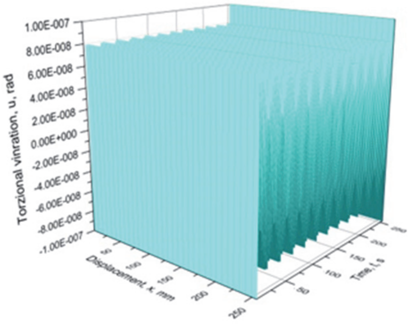

Figure 3: Torsional vibrations of the shaft for the excitation function u(x,t)= a_m*sin(x)

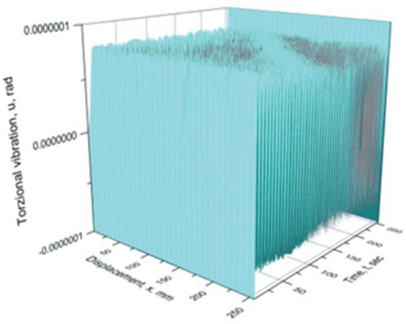

Figure 4: Torsional vibrations of the shaft for the excitation function u(x,t)= a_m*sin(x t)

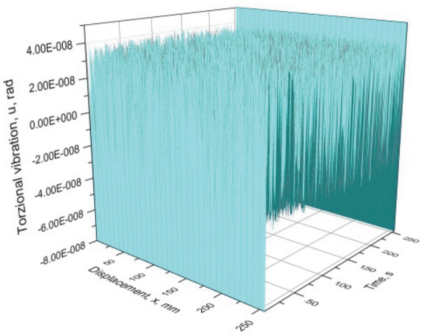

Figure 5: Torsional vibrations of the shaft for the excitation function u(x,t)=a_m*sin(rnd-0.5)

By analyzing figures 2-5, it can be observed that torsional vibrations have a harmonic character that travels in the form of waves along the length of the shaft, which is following the theoretical solutions from [13,14]. When excitation functions are harmonic functions, the character of torsional vibrations is more regular, while for random excitations, waves are less noticeable because the superposition of multiple waves with different wavelengths occurs.

Torsional vibrations of an elastic shaft depend on displacement x and time t, so a 2D Fourier transformation must be applied. The author developed software in Pascal for its realization. However, considering the available software on the market, it is considered appropriate to use Origin 8.5 [19] for further analysis, as potential users will have easier access to that software.

Using the mentioned software, the spectra magnitude and the phase of the two-parameter Fourier transform were calculated, and the results are shown in images 6-13.

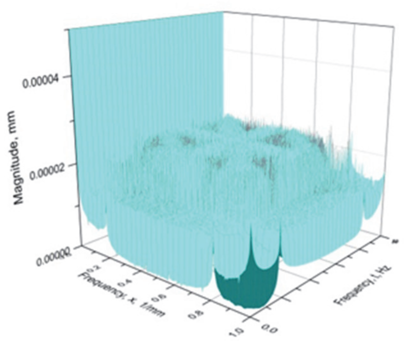

Figure 6: Spectra magnitude of torsional vibrations of the shaft for the excitation u(x,t)= a_m*sin(t)

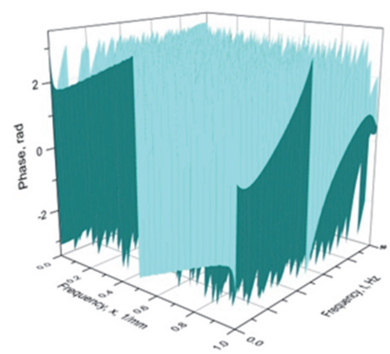

Figure 7: The phase angle of torsional vibrations of the shaft for the excitation u(x,t)= a_m*sin(t)

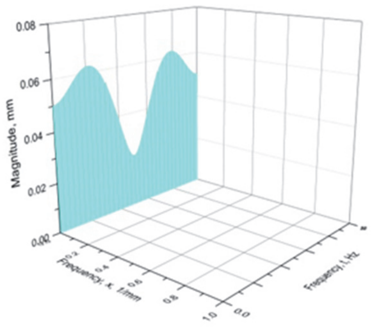

Figure 8: Spectra magnitude of torsional vibrations of the shaft for the excitation u(x,t)= a_m*sin(x)

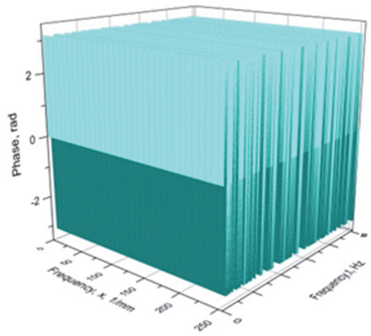

Figure 9: The phase angle of torsional vibrations of the shaft for the excitation u(x,t)= am*sin(x)

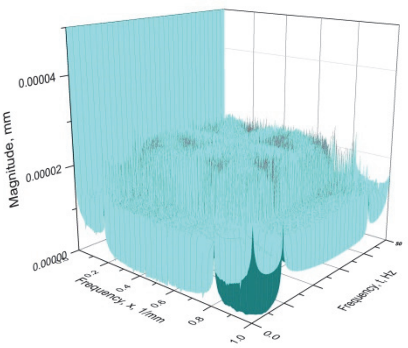

Figure 10: Spectra magnitude of torsional vibrations of the shaft for the excitation u(x,t)= a_m*sin(x t)

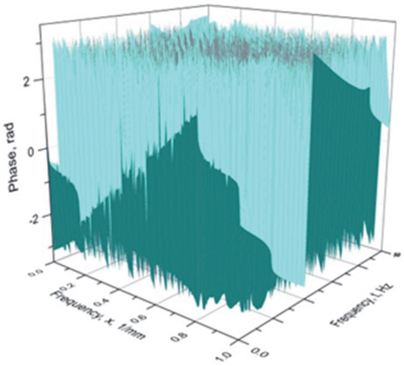

Figure 11: The phase angle of torsional vibrations of the shaft for the excitation u(x,t)= a_m*sin(x t)

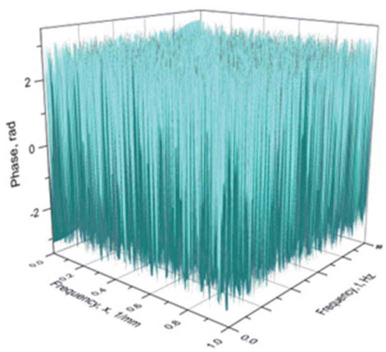

Figure 12: Spectra magnitude of torsional vibrations of the shaft for the excitation u(x,t)= a_m*(rnd-0.5)

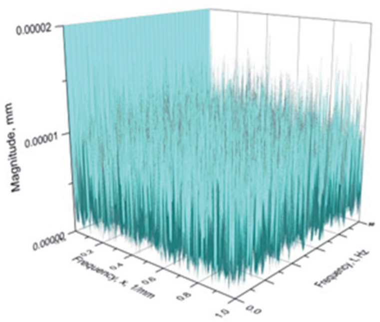

Figure 13: The phase angle of torsional vibrations of the shaft for the excitation u(x,t)= am*(rnd-0.5)

3. DATA ANALYSIS

Analysis of the data presented in Figures 6-13 leads to the conclusion that the spectra magnitude and phase angles identify the character of torsional vibrations of the elastic shaft. The solution of the partial differential equation (2) and the data obtained from the spectra magnitude and phase of the 2D Fourier transform show the wave-like character of the vibrations, which is following [13,14]. The waves are more noticeable in cases of harmonic forms of excitation functions, such as sin(t) and sin(x), which also holds partially for sin(xt), while in the case of random excitation, the waves are chaotic, as expected.

Based on the previous analyses, it can be concluded that the two-parameter Fourier transform reliably enable the analysis of data on torsional vibrations of the elastic shaft, which can have practical applications. The calculated spectra magnitude and phase angles, using the two-parameter inverse Fourier transform, allow for the generation of identical vibrations in the laboratory as those registered under operational conditions [19].

It should be noted that there are no explicit procedures for calculating spectral analysis errors in two-parameter Fourier transforms, as in the case of 1D Fourier transforms [11]. Considering this and the fact that this study is only aimed at illustrating the potential application of two-parameter frequency analysis in investigating torsional vibrations of an elastic shaft of vehicles, statistical error analysis was not performed.

It is worth noting that the inverse Fourier transform can be performed using the mentioned software, Origin 8.5 [19].

In the end, it should be noted that experimentally obtained data, processed by the 2D Fourier transformation, can be used in the laboratory to generate signals identical to those registered in exploitation conditions. During exploitation tests, it is necessary to record parameters of torsional vibrations of the elastic shaft (stress, angular displacement, speed or acceleration, and so on..) along its length, over a longer time. The selection of sensor placement steps is related to the minimum and maximum values of the frequency content. Namely, the minimum and maximum values of these parameters depend on the length of the shaft, i.e. the length of the time signal and the sample step.

First, the maximum interesting frequencies f_{xmax} and f_{tmax} should be adopted, and then the step of placing the sensor and sampling the time signal is defined based on the expression (Nyquist frequency) [11]:

$$f_{x\min} = \frac{1}{L} \qquad f_{t\min} = \frac{1}{T}$$

The minimum interesting frequency is obtained based on the length of the shaft ($L=n_x*h_x$), i.e. the length of the time signal ($T=n_t*h_t$) according to the expressions:

$$h_x = \frac{1}{2f_{x\max}} \qquad h_t = \frac{1}{2f_{t\max}}$$

Finally, it should be emphasized that the developed procedure has created conditions for analyzing the influence of integration steps on the accuracy and stability of solutions to the partial differential equation (2), the influence of design parameters on torsional vibrations of the elastic shaft, the influence of excitation torques, etc. However, given that the results of dynamic simulation in this study should only serve as a substitute for missing experimental results, it was estimated that a more detailed analysis is not necessary.

4. CONCLUSION

Based on the conducted research, it can be stated that the two-parameter Fourier transform reliably enables the analysis of experimental data of torsional vibrations of an elastic shaft. Calculated spectra magnitude and phase angles, with the application of the inverse 2D Fourier transform, enable the generation of identical vibrations in the laboratory as well as in operating conditions.

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